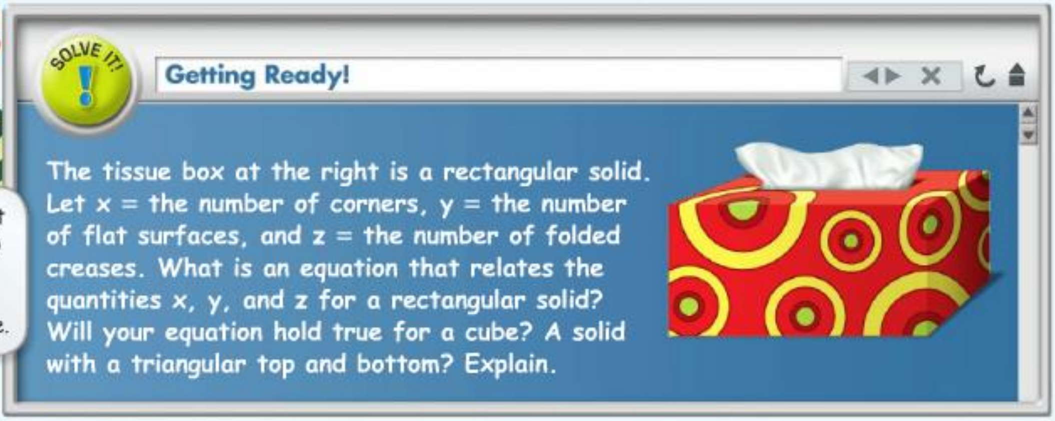
11-1 Space Figures and Cross Sections

Objectives To recognize polyhedra and their parts
To visualize cross sections of space figures

If you can reflect or rotate a net to get another net, then those two nets are the same.



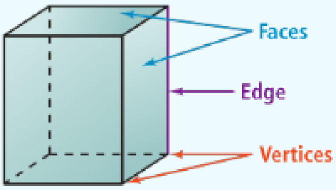
Getting Ready!

The tissue box at the right is a rectangular solid. Let x = the number of corners, y = the number of flat surfaces, and z = the number of folded creases. What is an equation that relates the quantities x, y, and z for a rectangular solid? Will your equation hold true for a cube? A solid with a triangular top and bottom? Explain.

Lesson Vocabulary
- polyhedron
- face
- edge
- vertex
- cross section

In the Solve It, you used two-dimensional nets to represent a three-dimensional object.

A **polyhedron** is a space figure, or three-dimensional figure, whose surfaces are polygons. Each polygon is a **face** of the polyhedron. An **edge** is a segment that is formed by the intersection of two faces. A **vertex** is a point where three or more edges intersect.

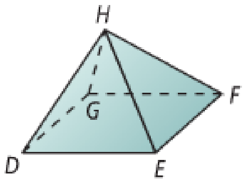


Essential Understanding You can analyze a three-dimensional figure by using the relationships among its vertices, edges, and faces.

Plan

Can you see the solid?
A dashed line indicates an edge that is hidden from view. This figure has one four-sided face and four triangular faces.

Problem 1 Identifying Vertices, Edges, and Faces

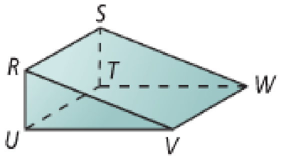


How many vertices, edges, and faces are in the polyhedron at the right? List them.

There are five vertices: *D*, *E*, *F*, *G*, and *H*.

There are eight edges: $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GD}$, $\overline{DH}$, $\overline{EH}$, $\overline{FH}$, and $\overline{GH}$.

There are five faces: $\triangle DEH$, $\triangle EFH$, $\triangle FGH$, $\triangle GDH$, and quadrilateral *DEFG*.



Got It? **1. a.** How many vertices, edges, and faces are in the polyhedron at the right? List them.

b. **Reasoning** Is $\overline{TV}$ an edge? Explain why or why not.

Leonhard Euler, a Swiss mathematician, discovered a relationship among the numbers of faces, vertices, and edges of any polyhedron. The result is known as Euler's Formula.

take note

Key Concept Euler's Formula

The sum of the number of faces (F) and vertices (V) of a polyhedron is two more than the number of its edges (E).

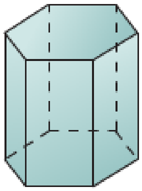
$$F + V = E + 2$$

Plan

How do you verify Euler's Formula? Find the number of faces, vertices, and edges. Then substitute the values into Euler's Formula to make sure that the equation is true.

Problem 2 Using Euler's Formula

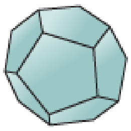
How many vertices, edges, and faces does the polyhedron at the right have? Use your results to verify Euler's Formula.



Think	Write
Count the number of faces.	$F = 8$
Count the number of vertices.	$V = 12$
Count the number of edges.	$E = 18$
Substitute the values into Euler's Formula.	$F + V = E + 2$ $8 + 12 \overset{?}{=} 18 + 2$ $20 = 20$ ✓

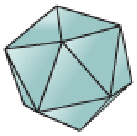
Got It? 2. For each polyhedron, use Euler's Formula to find the missing number.

a.



faces: ■
edges: 30
vertices: 20

b.



faces: 20
edges: ■
vertices: 12

In two dimensions, Euler's Formula reduces to $F + V = E + 1$, where F is the number of regions formed by V vertices linked by E segments.

Plan

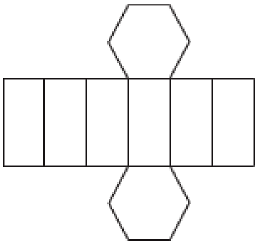
What do you use for the variables?

In 3-D → In 2-D
F: Faces → Regions
V: Vertices → Vertices
E: Edges → Segments

Problem 3 Verifying Euler's Formula in Two Dimensions

How can you verify Euler's Formula for a net for the solid in Problem 2?

Draw a net for the solid.



Number of regions: $F = 8$

Number of vertices: $V = 22$

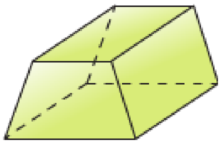
Number of segments: $E = 29$

$F + V = E + 1$ Euler's Formula for two dimensions

$8 + 22 = 29 + 1$ Substitute.

$30 = 30$ ✓

Got It? **3.** Use the solid at the right.

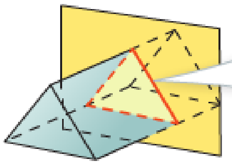


a. How can you verify Euler's Formula $F + V = E + 2$ for the solid?

b. Draw a net for the solid.

c. How can you verify Euler's Formula $F + V = E + 1$ for your two-dimensional net?

A **cross section** is the intersection of a solid and a plane. You can think of a cross section as a very thin slice of the solid.



This cross section is a triangle.

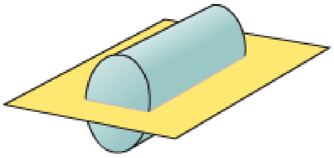
Think

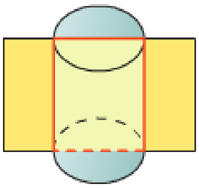
How can you see the cross section?

Mentally rotate the solid so that the plane is parallel to your face.

Problem 4 Describing a Cross Section

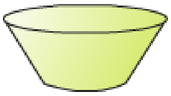
What is the cross section formed by the plane and the solid at the right?





The cross section is a rectangle.

Got It? **4.** For the solid at the right, what is the cross section formed by each of the following planes?

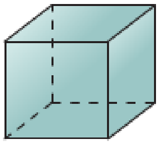


a. a horizontal plane

b. a vertical plane that divides the solid in half

To draw a cross section, you can sometimes use the idea from Postulate 1-3 that the intersection of two planes is exactly one line.

Problem 5 Drawing a Cross Section



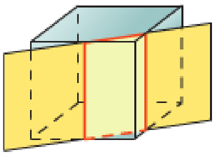
Visualization **Draw a cross section formed by a vertical plane intersecting the front and right faces of the cube. What shape is the cross section?**

Think

How can you see parallel segments? Focus on the plane intersecting the front and right faces. The plane and both faces are vertical, so the intersections are vertical parallel lines.

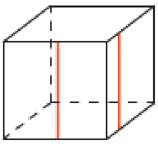
Step 1

Visualize a vertical plane intersecting the vertical faces in parallel segments.



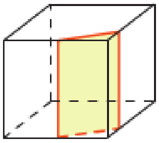
Step 2

Draw the parallel segments.



Step 3

Join their endpoints. Shade the cross section.



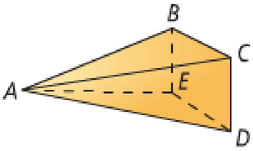
The cross section is a rectangle.

Got It? **5.** Draw the cross section formed by a horizontal plane intersecting the left and right faces of the cube. What shape is the cross section?

Lesson Check

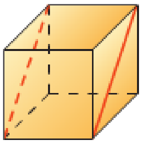
Do you know HOW?

1. How many faces, edges, and vertices are in the solid? List them.



2. What is a net for the solid in Exercise 1? Verify Euler's Formula for the net.

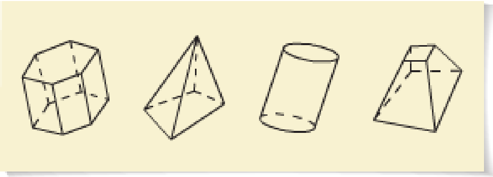
3. What is the cross section formed by the cube and the plane containing the diagonals of a pair of opposite faces?



Do you UNDERSTAND?

4. **Vocabulary** Suppose you build a polyhedron from two octagons and eight squares. Without using Euler's Formula, how many edges does the solid have? Explain.

5. **Error Analysis** Your math class is drawing polyhedrons. Which figure does not belong in the diagram below? Explain.



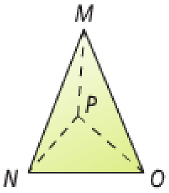
Practice and Problem-Solving Exercises

A Practice

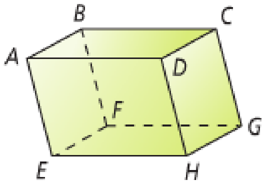
For each polyhedron, how many vertices, edges, and faces are there? List them.

See Problem 1.

6.



7.



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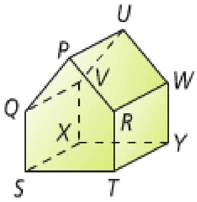


For each polyhedron, use Euler's Formula to find the missing number.

See Problem 2.

9. faces: ■
edges: 15
vertices: 9

10. faces: 8
edges: ■
vertices: 6

11. faces: 20
edges: 30
vertices: ■

Use Euler's Formula to find the number of vertices in each polyhedron.

12. 6 square faces

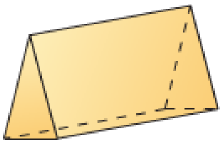
13. 5 faces: 1 rectangle and 4 triangles

14. 9 faces: 1 octagon and 8 triangles

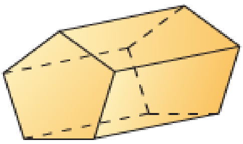
Verify Euler's Formula for each polyhedron. Then draw a net for the figure and verify Euler's Formula for the two-dimensional figure.

See Problem 3.

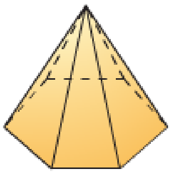
15.



16.



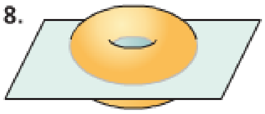
17.



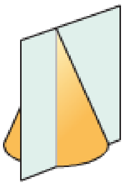
Describe each cross section.

See Problem 4.

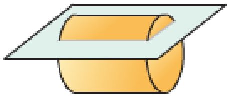
18.



19.



20.



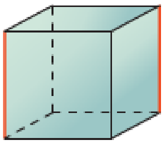
Visualization **Draw and describe a cross section formed by a vertical plane intersecting the cube as follows.**

See Problem 5.

21. The vertical plane intersects the front and left faces of the cube.

22. The vertical plane intersects opposite faces of the cube.

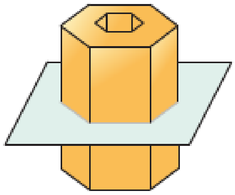
23. The vertical plane contains the red edges of the cube.



B Apply

24. **a.** **Open-Ended** Sketch a polyhedron whose faces are all rectangles. Label the lengths of its edges.

b. Use graph paper to draw two different nets for the polyhedron.



25. For the figure shown at the right, sketch each of following.

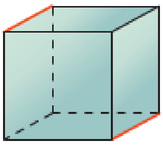
a. a horizontal cross section

b. a vertical cross section that contains the vertical line of symmetry

26. **Reasoning** Can you find a cross section of a cube that forms a triangle? Explain.

27. **Reasoning** Suppose the number of faces in a certain polyhedron is equal to the number of vertices. Can the polyhedron have nine edges? Explain.

Visualization **Draw and describe a cross section formed by a plane intersecting the cube as follows.**



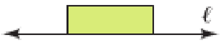
28. The plane is tilted and intersects the left and right faces of the cube.

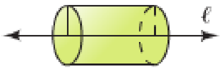
29. The plane contains the red edges of the cube.

30. The plane cuts off a corner of the cube.

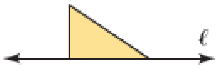
Visualization **A plane region that revolves completely about a line sweeps out a solid of revolution. Use the sample to help you describe the *solid of revolution* you get by revolving each region about line ℓ.**

Sample: Revolve the rectangular region about the line ℓ. You get a cylinder as the solid of revolution.

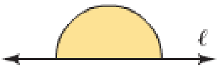




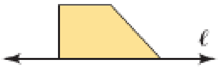
31.



32.



33.

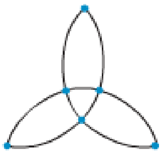


34. **Think About a Plan** Some balls are made from panels that suggest polygons. A soccer ball suggests a polyhedron with 20 regular hexagons and 12 regular pentagons. How many vertices does this polyhedron have?

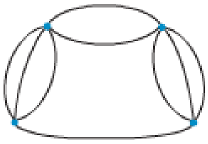
- How can you determine the number of edges in a solid if you know the types of polygons that form the faces?
- What relationship can you use to find the number of vertices?

Euler's Formula $F + V = E + 1$ applies to any two-dimensional network where F is the number of regions formed by V vertices linked by E edges (or paths). Verify Euler's Formula for each network shown.

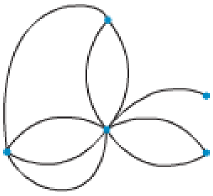
35.



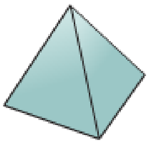
36.

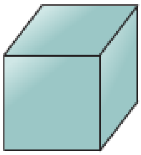


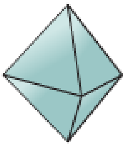
37.

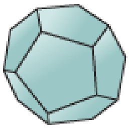


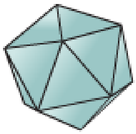
38. Platonic Solids There are five regular polyhedrons. They are called *regular* because all their faces are congruent regular polygons, and the same number of faces meet at each vertex. They are also called *Platonic solids* after the Greek philosopher Plato, who first described them in his work *Timaeus* (about 350 B.C.).


Tetrahedron

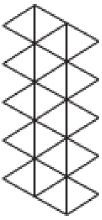

Hexahedron

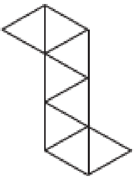

Octahedron

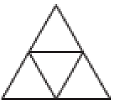

Dodecahedron

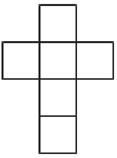

Icosahedron

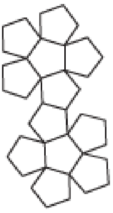
a. Match each net below with a Platonic solid.

A.


B.


C.


D.


E.


b. The first two Platonic solids have more familiar names. What are they?

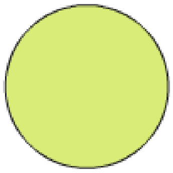
c. Verify that Euler's Formula is true for the first three Platonic solids.

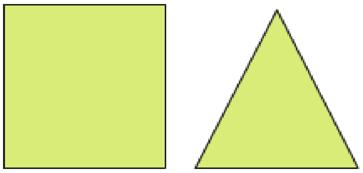
39. A cube has a net with area 216 in.^{2}. How long is an edge of the cube?

40. Writing Cross sections are used in medical training and research. Research and write a paragraph on how magnetic resonance imaging (MRI) is used to study cross sections of the brain.

Challenge

41. Open-Ended Draw a single solid that has the following three cross sections.


Horizontal


Vertical

Visualization **Draw a plane intersecting a cube to get the cross section indicated.**

42. scalene triangle

43. isosceles triangle

44. equilateral triangle

45. trapezoid

46. isosceles trapezoid

47. parallelogram

48. rhombus

49. pentagon

50. hexagon

Standardized Test Prep

SAT/ACT

51. A polyhedron has four vertices and six edges. How many faces does it have?

Ⓐ 2 Ⓑ 4 Ⓒ 5 Ⓓ 10

52. Suppose the circumcenter of $\triangle ABC$ lies on one of its sides. What type of triangle must $\triangle ABC$ be?

Ⓕ scalene Ⓖ isosceles Ⓗ equilateral Ⓘ right

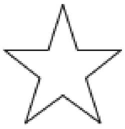
53. What is the area of a regular hexagon whose perimeter is 36 in.?

Ⓐ $18\sqrt{3}$ in.^{2} Ⓑ $27\sqrt{3}$ in.^{2} Ⓒ $36\sqrt{3}$ in.^{2} Ⓓ $54\sqrt{3}$ in.^{2}

54. What is the best description of the polygon at the right?

Ⓕ concave decagon Ⓗ regular pentagon

Ⓖ convex decagon Ⓘ regular decagon



Short Response

55. The coordinates of three vertices of a parallelogram are $A(2, 1)$, $B(1, -2)$ and $C(4, -1)$. What are the coordinates of the fourth vertex D? Explain.

Mixed Review

56. Probability Shuttle buses to an airport terminal leave every 20 min from a remote parking lot. Draw a geometric model and find the probability that a traveler who arrives at a random time will have to wait at least 8 min for the bus to leave the parking lot.

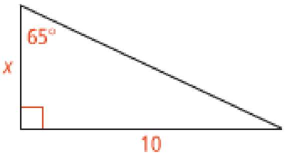
See Lesson 10-8.

57. Games A dartboard is a circle with a 12-in. radius. What is the probability that you throw a dart that lands within 6 in. of the center of the dartboard?

Find the value of *x* to the nearest tenth.

See Lesson 8-3.

58.



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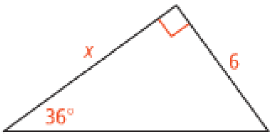
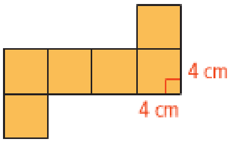


Get Ready! **To prepare for Lesson 11-2, do Exercises 60–62.**

Find the area of each net.

See Lessons 1-8 and 10-3.

60.



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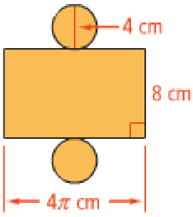

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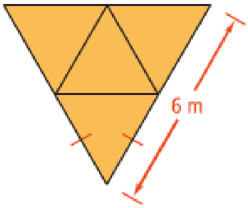


Concept Byte

Use With Lesson 11-1

EXTENSION

Perspective Drawing

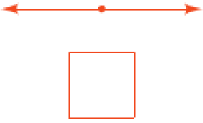
You can draw a three-dimensional space figure using a two-dimensional *perspective drawing*. Suppose two lines are parallel in three dimensions, but extend away from the viewer. You draw them—and create perspective—so that they meet at a *vanishing point* on a *horizon line*.

Example 1

Draw a cube in one-point perspective.

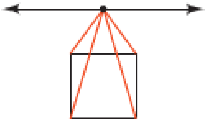
Step 1

Draw a square. Then draw a horizon line and a vanishing point on the line.



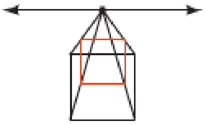
Step 2

Lightly draw segments from the vertices of the square to the vanishing point.



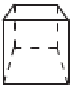
Step 3

Draw a square for the back of the cube. Each vertex should lie on a segment you drew in Step 2.



Step 4

Complete the figure by using dashes for the hidden edges of the cube. Erase unneeded lines.

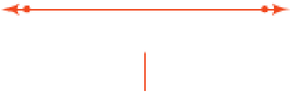


Example 2

Draw a box in two-point perspective.

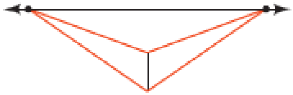
Step 1

Draw a vertical segment. Then draw a horizon line and two vanishing points on the line.



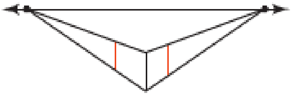
Step 2

Lightly draw segments from the endpoints of the vertical segment to each vanishing point.



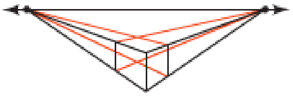
Step 3

Draw two vertical segments between the segments of Step 2.



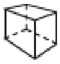
Step 4

Draw segments from the endpoints of the segments you drew in Step 3 to the vanishing points.



Step 5

Complete the figure by using dashes for the hidden edges of the figure. Erase unneeded lines.

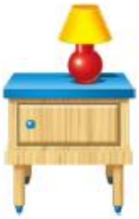


In one-point perspective, the front of the cube is parallel to the drawing surface. A two-point perspective drawing generally looks like a corner view. For either type of drawing, you should be able to envision each vanishing point.

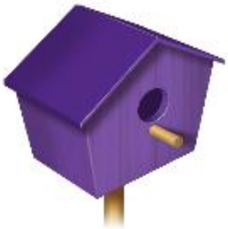
Exercises

Is each object drawn in one-point or two-point perspective?

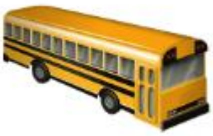
1.



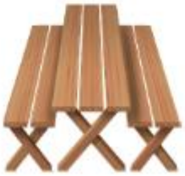
2.



3.



4.



Draw each object in one-point perspective and then in two-point perspective.

5. a shoe box

6. a building in your town that sits on a street corner

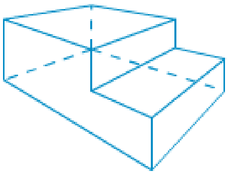
Draw each container using one-point perspective. Show a base at the front.

7. a triangular carton

8. a hexagonal box

Copy each figure and locate the vanishing point(s).

9.



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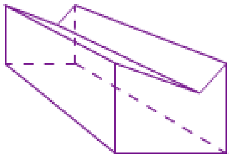
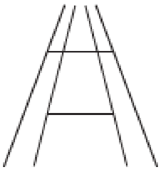
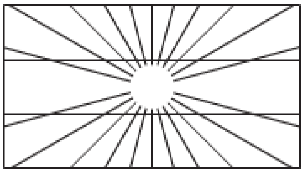


Optical Illusions **What is the optical illusion? Explain the role of perspective in each illusion.**

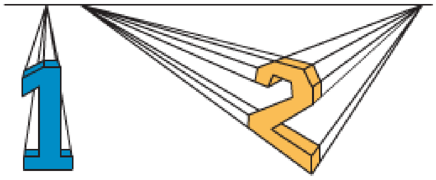
11.



12.



13. **Open-Ended** You can draw block letters in either one-point or two-point perspective. Write your initials in block letters using one-point perspective and two-point perspective.



Algebra Review

Use With Lesson 11-2

Literal Equations

A *literal equation* is an equation involving two or more variables. A formula is a special type of literal equation. You can transform a formula by solving for one variable in terms of the others.

Example

A The formula for the volume of a cylinder is $V = \pi r^2 h$. Find a formula for the height in terms of the radius and volume.

$V = \pi r^2 h$ Use the formula for the volume of a cylinder.

$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$ Divide each side by πr^2, with $r \neq 0$.

$\frac{V}{\pi r^2} = h$ Simplify.

The formula for the height is $h = \frac{V}{\pi r^2}$.

B Find a formula for the area of a square in terms of its perimeter.

$P = 4s$ Use the formula for the perimeter of a square.

$\frac{P}{4} = s$ Solve for s in terms of P.

$A = s^2$ Use the formula for area.

$= \left(\frac{P}{4}\right)^2$ Substitute $\frac{P}{4}$ for s.

$= \frac{P^2}{16}$ Simplify.

The formula for the area is $A = \frac{P^2}{16}$.

Exercises

Algebra **Solve each equation for the variable in red.**

1. $C = 2\pi r$

2. $A = \frac{1}{2}bh$

3. $A = \pi r^2$

Algebra **Solve for the variable in red. Then solve for the variable in blue.**

4. $P = 2w + 2\ell$

5. $\tan A = \frac{y}{x}$

6. $A = \frac{1}{2}(b_1 + b_2)h$

Find a formula as described below.

7. the circumference C of a circle in terms of its area A

8. the area A of an isosceles right triangle in terms of the hypotenuse h

9. the apothem a of a regular hexagon in terms of the area A of the hexagon

10. Solve $A = \frac{1}{2}ab \sin C$ for $m\angle C$.

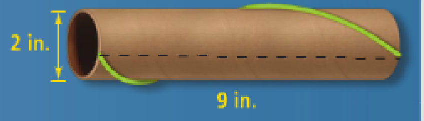
11-2 Surface Areas of Prisms and Cylinders

Objective To find the surface area of a prism and a cylinder

You'll flatten this problem out in no time!

SOLVE IT! **Getting Ready!**

A piece of string is wrapped once around an empty paper towel tube. The ends of the string are attached to each end of the tube as shown. How long is the piece of string? Justify your reasoning.

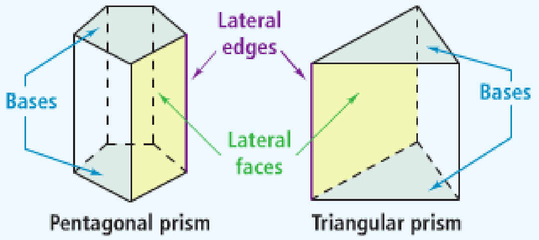


Lesson Vocabulary
- prism (base, lateral face, altitude, height, lateral area, surface area)
- right prism
- oblique prism
- cylinder (base, altitude, height, lateral area, surface area)
- right cylinder
- oblique cylinder

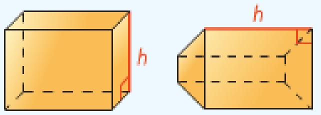
In the Solve It, you investigated the structure of a tube. In this lesson, you will learn properties of three-dimensional figures by investigating their surfaces.

Essential Understanding To find the surface area of a three-dimensional figure, find the sum of the areas of all the surfaces of the figure.

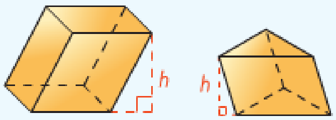
A **prism** is a polyhedron with two congruent, parallel faces, called **bases.** The other faces are **lateral faces.** You can name a prism using the shape of its bases.



An **altitude** of a prism is a perpendicular segment that joins the planes of the bases. The **height** h of a prism is the length of an altitude. A prism may either be right or oblique.



Right prisms



Oblique prisms

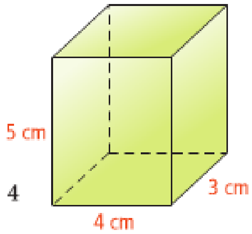
In a **right prism,** the lateral faces are rectangles and a lateral edge is an altitude. In an **oblique prism,** some or all of the lateral faces are nonrectangular. In this book, you may assume that a prism is a right prism unless stated or pictured otherwise.

The **lateral area** (L.A.) of a prism is the sum of the areas of the lateral faces. The **surface area** (S.A.) is the sum of the lateral area and the area of the two bases.

Think

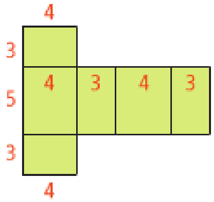
How do you know what units to use?
In the prism, the rectangle marked 5 on one side and 4 on the other has area 5 cm × 4 cm, or 20 cm^2. So use cm^2 as the unit for the surface area of the prism.

Problem 1 Using a Net to Find Surface Area of a Prism



What is the surface area of the prism at the right? Use a net.

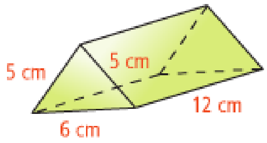
Draw a net for the prism. Then calculate the surface area.



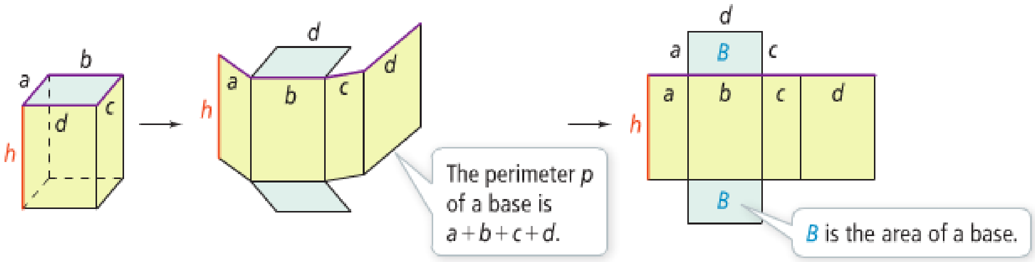
$$\begin{aligned} \text{S.A.} &= \text{sum of areas of all the faces} \\ &= 5 \cdot 4 + 5 \cdot 3 + 5 \cdot 4 + 5 \cdot 3 + 3 \cdot 4 + 3 \cdot 4 \\ &= 20 + 15 + 20 + 15 + 12 + 12 \\ &= 94 \end{aligned}$$

The surface area of the prism is 94 cm^2.

Got It? **1.** What is the surface area of the triangular prism? Use a net.



You can find formulas for lateral and surface areas of a prism by using a net.



The perimeter p of a base is $a+b+c+d$.

B is the area of a base.

Lateral Area $= ph$

Surface Area $=$ L.A. $+ 2B$

You can use the formulas with any right prism.

Dynamic Activity
Surface and Lateral Area of Prisms and Cylinders

take note

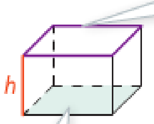
Theorem 11-1 Lateral and Surface Areas of a Prism

The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

$$\text{L.A.} = ph$$

The surface area of a right prism is the sum of the lateral area and the areas of the two bases.

$$\text{S.A.} = \text{L.A.} + 2B$$

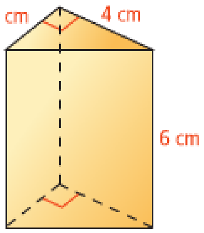


p is the perimeter of a base.

B is the area of a base.

Problem 2 Using Formulas to Find Surface Area of a Prism

What is the surface area of the prism at the right?



Plan

What do you need to find first?
You first need to find the missing side length of a triangular base so that you can find the perimeter of a base.

Step 1 Find the perimeter of a base.

The perimeter of the base is the sum of the side lengths of the triangle. Since the base is a right triangle, the hypotenuse is $\sqrt{3^2 + 4^2}$ cm, or 5 cm, by the Pythagorean Theorem.

$p = 3 + 4 + 5 = 12$

Step 2 Find the lateral area of the prism.

$\text{L.A.} = ph$ Use the formula for lateral area.

$= 12 \cdot 6$ Substitute 12 for p and 6 for h.

$= 72$ Simplify.

Think

Which height do you need?
For problems involving solids, make it a habit to note which height the formula requires. In Step 3, you need the height of the triangle, not the height of the prism.

Step 3 Find the area of a base.

$B = \frac{1}{2}bh$ Use the formula for the area of a triangle.

$= \frac{1}{2}(3 \cdot 4)$ Substitute 3 for b and 4 for h.

$= 6$

Step 4 Find the surface area of the prism.

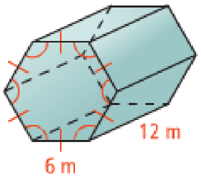
$\text{S.A.} = \text{L.A.} + 2B$ Use the formula for surface area.

$= 72 + 2(6)$ Substitute 72 for L.A. and 6 for B.

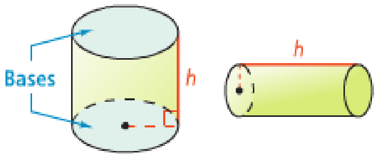
$= 84$ Simplify.

The surface area of the prism is 84 cm^2.

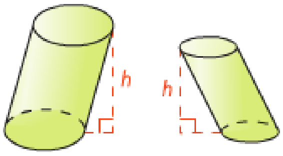
Got It? **2. a.** What is the lateral area of the prism at the right?
b. What is the area of a base in simplest radical form?
c. What is the surface area of the prism rounded to a whole number?



A **cylinder** is a solid that has two congruent parallel **bases** that are circles. An **altitude** of a cylinder is a perpendicular segment that joins the planes of the bases. The **height** h of a cylinder is the length of an altitude.



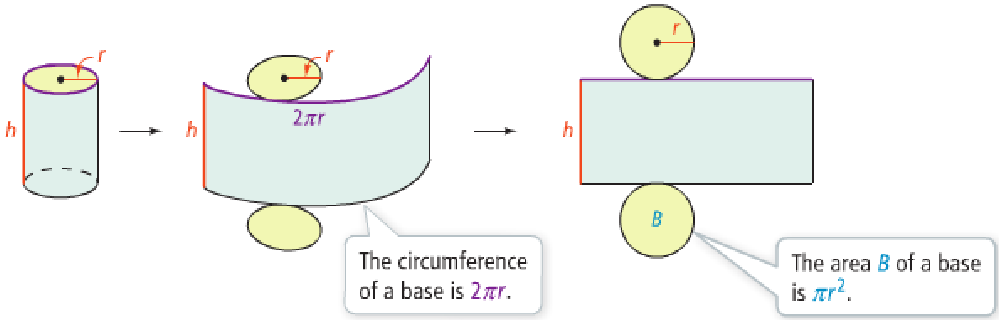
Right cylinders



Oblique cylinders

In a **right cylinder,** the segment joining the centers of the bases is an altitude. In an **oblique cylinder,** the segment joining the centers is not perpendicular to the planes containing the bases. In this book, you may assume that a cylinder is a right cylinder unless stated or pictured otherwise.

To find the area of the curved surface of a cylinder, visualize "unrolling" it. The area of the resulting rectangle is the **lateral area** of the cylinder. The **surface area** of a cylinder is the sum of the lateral area and the areas of the two circular bases. You can find formulas for these areas by looking at a net for a cylinder.



The circumference of a base is $2\pi r$.

The area B of a base is πr^2.

Lateral Area $= 2\pi rh$ Surface Area $=$ L.A. $+ 2\pi r^2$

take note

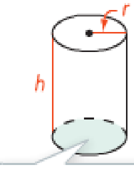
Theorem 11-2 Lateral and Surface Areas of a Cylinder

The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.

$\text{L.A.} = 2\pi r \cdot h$, or $\text{L.A.} = \pi dh$

The surface area of a right cylinder is the sum of the lateral area and the areas of the two bases.

$\text{S.A.} = \text{L.A.} + 2B$, or $\text{S.A.} = 2\pi rh + 2\pi r^2$



B is the area of a base.

Problem 3 Finding Surface Area of a Cylinder

Think

How is finding the surface area of a cylinder like finding the surface area of a prism?
For both, you need to find the L.A. and add it to twice the area of a base.

Multiple Choice The radius of the base of a cylinder is 4 in. and its height is 6 in. What is the surface area of the cylinder in terms of π?

Ⓐ 32π in.^{2} Ⓑ 42π in.^{2} Ⓒ 80π in.^{2} Ⓓ 120π in.^{2}

$\text{S.A.} = \text{L.A.} + 2B$	Use the formula for surface area of a cylinder.
$= 2\pi rh + 2(\pi r^2)$	Substitute the formulas for lateral area and area of a circle.
$= 2\pi(4)(6) + 2(\pi 4^2)$	Substitute 4 for r and 6 for h.
$= 48\pi + 32\pi$	Simplify.
$= 80\pi$	

The surface area of the cylinder is 80π in.^{2}. The correct choice is C.

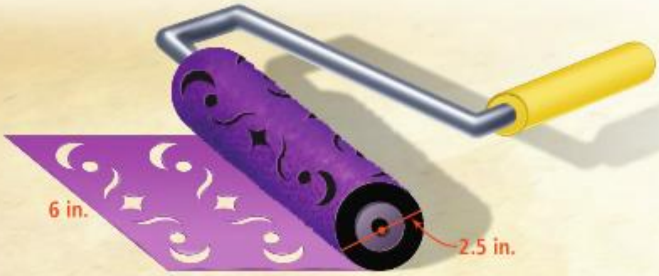
Got It? **3.** A cylinder has a height of 9 cm and a radius of 10 cm. What is the surface area of the cylinder in terms of π?

Think

What is the problem asking you to find?
The area covered by one full turn of the roller is the rectangular part of the net of a cylinder. This is the lateral area.

Problem 4 Finding Lateral Area of a Cylinder

Interior Design **You are using the cylindrical stencil roller below to paint patterns on your floor. What area does the roller cover in one full turn?**



The area covered is the lateral area of a cylinder with height 6 in. and diameter 2.5 in.

$\text{L.A.} = \pi d h$ Use the formula for lateral area of a cylinder.

$= \pi(2.5)(6)$ Substitute 2.5 for *d* and 6 for *h*.

$= 15\pi \approx 47.1$ Simplify.

In one full turn, the stencil roller covers about 47.1 in.^{2}.

Got It? **4. a.** A smaller stencil roller has a height of 1.5 in. and the same diameter as the roller in Problem 1. What area does the smaller roller cover in one turn? Round your answer to the nearest tenth.

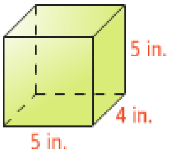
b. **Reasoning** What is the ratio of the smaller roller's height to the larger roller's height? What is the ratio of the areas the rollers can cover in one turn (smaller to larger)?

Lesson Check

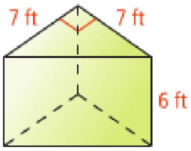
Do you know HOW?

What is the surface area of each prism?

1.

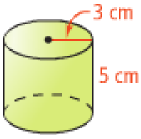


2.



What is the surface area of each cylinder?

3.



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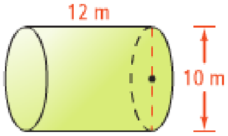
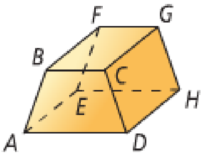
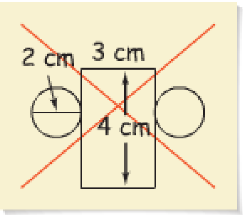


Do you UNDERSTAND?

5. **Vocabulary** Name the lateral faces and the bases of the prism at the right.



6. **Error Analysis** Your friend drew a net of a cylinder. What is your friend's error? Explain.

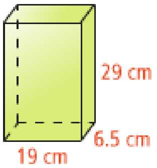


Practice and Problem-Solving Exercises

A Practice

Use a net to find the surface area of each prism. **See Problem 1.**

7.



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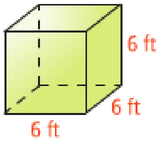
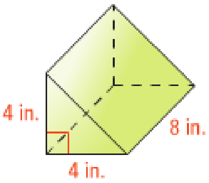


9.

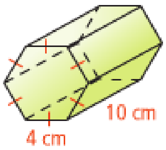


10. **a.** Classify the prism at the right.

b. Find the lateral area of the prism.

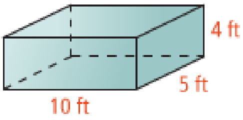
c. The bases are regular hexagons. Find the sum of their areas.

d. Find the surface area of the prism.

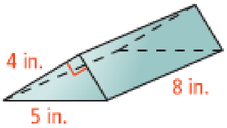


Use formulas to find the surface area of each prism. Round your answer to the nearest whole number. **See Problem 2.**

11.



12.



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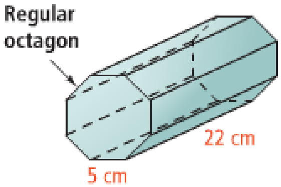
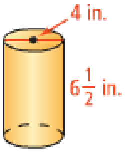


Find the lateral area of each cylinder to the nearest whole number. **See Problems 3 and 4.**

14.



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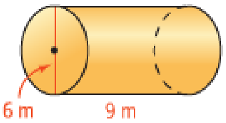

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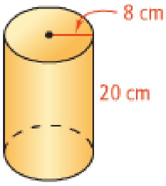
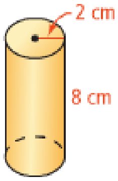


Find the surface area of each cylinder in terms of π.

17.



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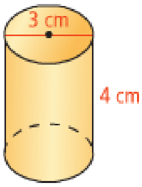
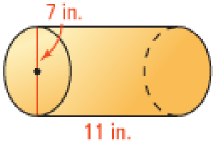


19.



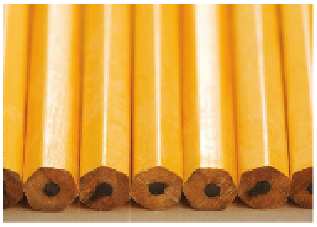
20. **Packaging** A cylindrical carton of oatmeal with radius 3.5 in. is 9 in. tall. If all surfaces except the top are made of cardboard, how much cardboard is used to make the oatmeal carton? Assume no surfaces overlap. Round your answer to the nearest square inch.

B Apply

21. A triangular prism has base edges 4 cm, 5 cm, and 6 cm long. Its lateral area is 300 cm^2. What is the height of the prism?

22. **Writing** Explain how a cylinder and a prism are alike and how they are different.

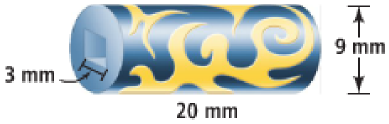
23. **Pencils** A hexagonal pencil is a regular hexagonal prism, as shown at the right. A base edge of the pencil has a length of 4 mm. The pencil (without eraser) has a height of 170 mm. What is the area of the surface of the pencil that gets painted?



24. **Open-Ended** Draw a net for a rectangular prism with a surface area of 220 cm^2.

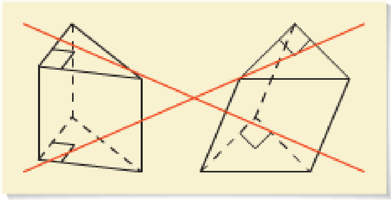
25. Consider a box with dimensions 3, 4, and 5.
- **a.** Find its surface area.
- **b.** Double each dimension and then find the new surface area.
- **c.** Find the ratio of the new surface area to the original surface area.
- **d.** Repeat parts (a)–(c) for a box with dimensions 6, 9, and 11.
- **e.** **Make a Conjecture** How does doubling the dimensions of a rectangular prism affect the surface area?

26. **Think About a Plan** A cylindrical bead has a square hole, as shown at the right. Find the area of the surface of the entire bead.
- Can you visualize the bead as a combination of familiar figures?
- How do you find the area of the surface of the inner part of the bead?

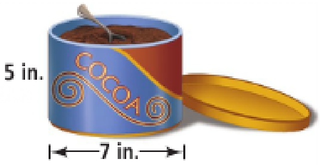


27. **Estimation** Estimate the surface area of a cube with edges 4.95 cm long.

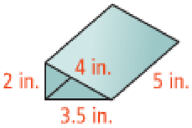
28. **Error Analysis** Your class is drawing right triangular prisms. Your friend's paper is below. What is your friend's error?



29. **Packaging** A cylindrical can of cocoa has the dimensions shown at the right. What is the approximate surface area available for the label? Round to the nearest square inch.

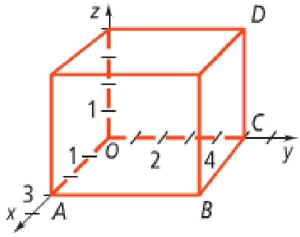

5 in.
COCOA
7 in.

30. **Pest Control** A flour moth trap has the shape of a triangular prism that is open on both ends. An environmentally safe chemical draws the moth inside the prism, which is lined with an adhesive. What is the area of the trap that is lined with the adhesive?

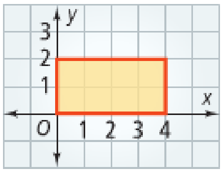


31. Suppose that a cylinder has a radius of r units, and that the height of the cylinder is also r units. The lateral area of the cylinder is 98π square units.
  a. Algebra Find the value of r.
  b. Find the surface area of the cylinder.

32. Geometry in 3 Dimensions Use the diagram at the right.
  a. Find the three coordinates of each vertex A, B, C, and D of the rectangular prism.
  b. Find AB.
  c. Find BC.
  d. Find CD.
  e. Find the surface area of the prism.



Visualization **Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid and find its surface area in terms of π.**



33. the y-axis

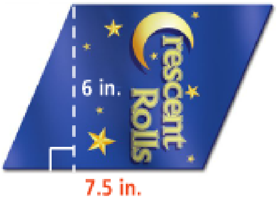
34. the x-axis

35. the line $y = 2$

36. the line $x = 4$

37. Reasoning Suppose you double the radius of a right cylinder.
  a. How does that affect the lateral area?
  b. How does that affect the surface area?
  c. Use the formula for surface area of a right cylinder to explain why the surface area in part (b) was not doubled.

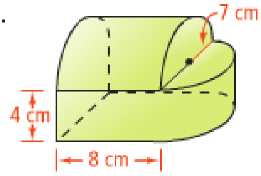
38. Packaging Some cylinders have wrappers with a spiral seam. Peeled off, the wrapper has the shape of a parallelogram. The wrapper for a biscuit container has base 7.5 in. and height 6 in.
  a. Find the radius and height of the container.
  b. Find the surface area of the container.



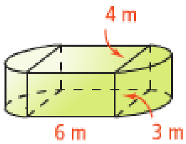
Challenge

What is the surface area of each solid in terms of π?

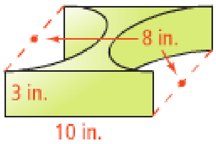
39.



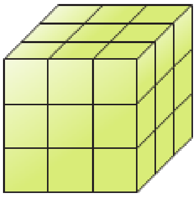
40.



41.



42. Each edge of the large cube at the right is 12 inches long. The cube is painted on the outside, and then cut into 27 smaller cubes. Answer these questions about the 27 cubes.
  a. How many are painted on 4, 3, 2, 1, and 0 faces?
  b. What is the total surface area that is unpainted?



43. Algebra The sum of the height and radius of a cylinder is 9 m. The surface area of the cylinder is 54π m^2. Find the height and the radius.

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

44. The height of a cylinder is twice the radius of the base. The surface area of the cylinder is 56π ft^2. What is the diameter of the base to the nearest tenth of a foot?

45. Two sides of a triangle measure 11 ft and 23 ft. What is the smallest possible whole number length, in feet, for the third side?

46. A polyhedron has one hexagonal face and six triangular faces. How many vertices does the polyhedron have?

47. The shortest shadow cast by a tree is 8 m long. The height of the tree is 20 m. To the nearest degree, what is the angle of elevation of the sun when the shortest shadow is cast?

Mixed Review

Sketch each space figure and then draw a net for it. Label the net with its dimensions.

See Lessons 1-1 and 11-1.

48. a rectangular prism with height 5 cm and a base 3 cm by 4 cm

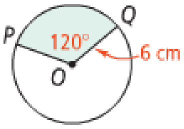
49. a cylinder with a 72π-in. circumference and a 22-in. height

Find the area of each part of the circle to the nearest tenth.

See Lesson 10-7.

50. sector QOP

51. the segment of the circle bounded by $\overline{QP}$ and $\overset{\frown}{QP}$



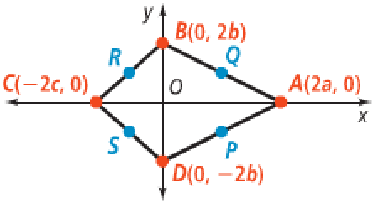
52. In the kite at the right, $AB = AD$ and $CB = CD$. Points P, Q, R, and S are midpoints.

See Lesson 6-7.

a. What are the coordinates of the midpoints?

b. $RQ = ■$; $SP = ■$; $PQ = ■$; $SR = ■$

c. Use your answers to part (b) to explain why $PQRS$ must be a parallelogram.

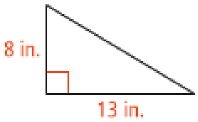


Get Ready! **To prepare for Lesson 11-3, do Exercises 53–55.**

Find the length of the hypotenuse in simplest radical form.

See Lesson 8-1.

53.



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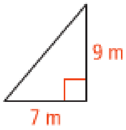

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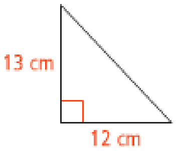
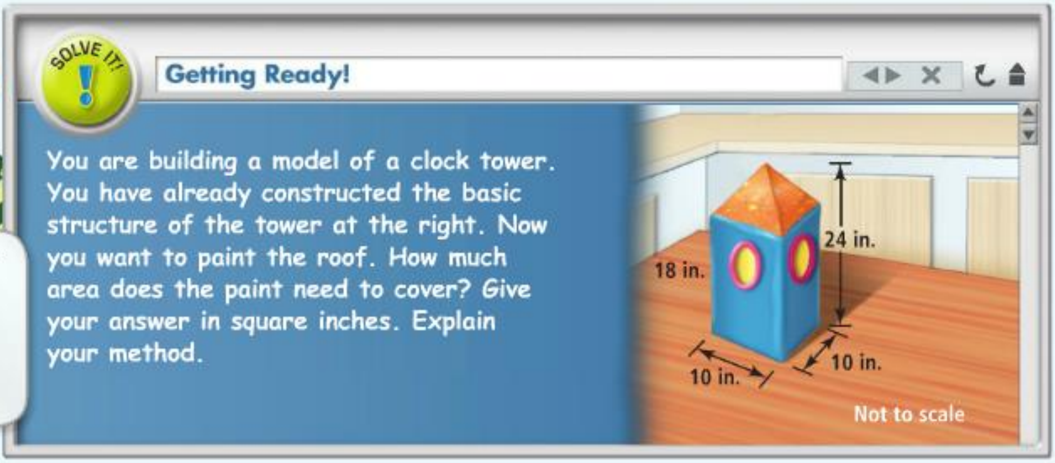


11-3 Surface Areas of Pyramids and Cones

Objective To find the surface area of a pyramid and a cone

Think about what dimensions you need and how to get them from what you already know.



Getting Ready!

You are building a model of a clock tower. You have already constructed the basic structure of the tower at the right. Now you want to paint the roof. How much area does the paint need to cover? Give your answer in square inches. Explain your method.

Lesson Vocabulary

- pyramid (base, lateral face, vertex, altitude, height, slant height, lateral area, surface area)
- regular pyramid
- cone (base, altitude, vertex, height, slant height, lateral area, surface area)
- right cone

The Solve It involves the triangular faces of a roof and the three-dimensional figures they form. In this lesson, you will learn to name such figures and to use formulas to find their areas.

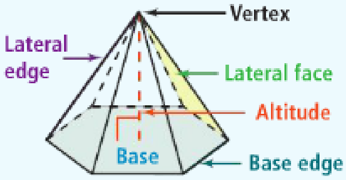
Essential Understanding To find the surface area of a three-dimensional figure, find the sum of the areas of all the surfaces of the figure.

A **pyramid** is a polyhedron in which one face (the **base**) can be any polygon and the other faces (the **lateral faces**) are triangles that meet at a common vertex (called the **vertex** of the pyramid).

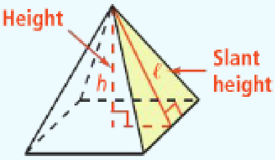
You name a pyramid by the shape of its base. The **altitude** of a pyramid is the perpendicular segment from the vertex to the plane of the base. The length of the altitude is the **height** h of the pyramid.

A **regular pyramid** is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles. The **slant height** ℓ is the length of the altitude of a lateral face of the pyramid.

In this book, you can assume that a pyramid is regular unless stated otherwise.



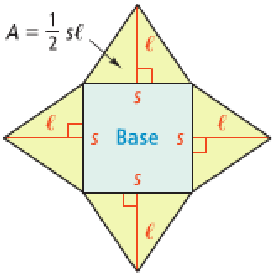
Hexagonal pyramid



Square pyramid

Dynamic Activity
Surface and Lateral Areas of Pyramids and Cones

The **lateral area** of a pyramid is the sum of the areas of the congruent lateral faces. You can find a formula for the lateral area of a pyramid by looking at its net.



$\text{L.A.} = 4\left(\frac{1}{2}s\ell\right)$ The area of each lateral face is $\frac{1}{2}s\ell$.

$= \frac{1}{2}(4s)\ell$ Commutative and Associative Properties of Multiplication

$= \frac{1}{2}p\ell$ The perimeter p of the base is $4s$.

To find the **surface area** of a pyramid, add the area of its base to its lateral area.

take note

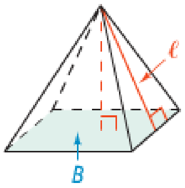
Theorem 11-3 Lateral and Surface Areas of a Pyramid

The lateral area of a regular pyramid is half the product of the perimeter p of the base and the slant height ℓ of the pyramid.

$$\text{L.A.} = \frac{1}{2}p\ell$$

The surface area of a regular pyramid is the sum of the lateral area and the area B of the base.

$$\text{S.A.} = \text{L.A.} + B$$

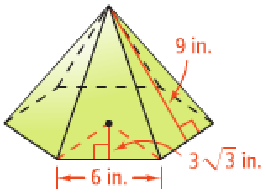


Problem 1 Finding the Surface Area of a Pyramid

Think

What is B?
B is the area of the base, which is a hexagon. You are given the apothem of the hexagon and the length of a side. Use them to find the area of the base.

What is the surface area of the hexagonal pyramid?



$\text{S.A.} = \text{L.A.} + B$ Use the formula for surface area.

$= \frac{1}{2}p\ell + \frac{1}{2}ap$ Substitute the formulas for L.A. and B.

$= \frac{1}{2}(36)(9) + \frac{1}{2}\left(3\sqrt{3}\right)(36)$ Substitute.

≈ 255.5307436 Use a calculator.

The surface area of the pyramid is about 256 in.^{2}.

Got It? **1. a.** A square pyramid has base edges of 5 m and a slant height of 3 m. What is the surface area of the pyramid?

b. Reasoning Suppose the slant height of a pyramid is doubled. How does this affect the lateral area of the pyramid? Explain.

When the slant height of a pyramid is not given, you must calculate it before you can find the lateral area or surface area.

Problem 2 Finding the Lateral Area of a Pyramid

Social Studies **The Pyramid of Cestius is located in Rome, Italy. Using the dimensions in the figure below, what is the lateral area of the Pyramid of Cestius? Round to the nearest square meter.**

Know

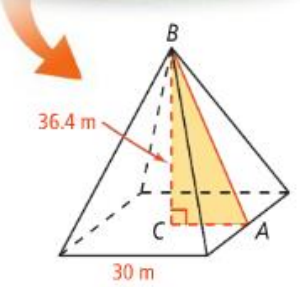
- The height of the pyramid
- The base is a square with a side length of 30 m.
- $\triangle ABC$ is right, where AB is the slant height.

Need

The slant height of the pyramid

Plan

Find the perimeter of the base. Use the Pythagorean Theorem to find the slant height. Then use the formula for lateral area.



Step 1 Find the perimeter of the base.

$p = 4s$ Use the formula for the perimeter of a square.

$= 4 \cdot 30$ Substitute 30 for s.

$= 120$ Simplify.

Think

Why is a new diagram helpful for finding the slant height?

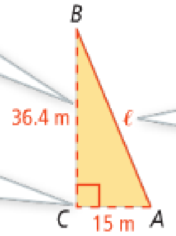
The new diagram shows the information you need to use the Pythagorean Theorem.

Step 2 Find the slant height of the pyramid.

BC is the height of the pyramid.

The slant height is the length of the hypotenuse of right $\triangle ABC$, or AB.

$\overline{CA}$ is the apothem of the base. Its length is $\frac{30}{2}$ m, or 15 m.



$\ell = \sqrt{CA^2 + BC^2}$ Use the Pythagorean Theorem.

$= \sqrt{15^2 + 36.4^2}$ Substitute 15 for CA and 36.4 for BC.

$= \sqrt{1549.96}$ Simplify.

Step 3 Find the lateral area of the pyramid.

$\text{L.A.} = \frac{1}{2}p\ell$ Use the formula for lateral area.

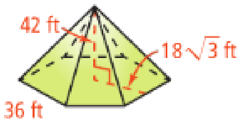
$= \frac{1}{2}(120)\sqrt{1549.96}$ Substitute 120 for p and $\sqrt{1549.96}$ for ℓ.

≈ 2362.171882 Use a calculator.

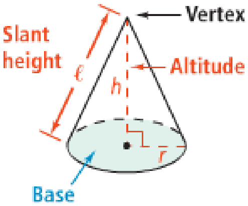
The lateral area of the Pyramid of Cestius is about 2362 m^2.

Got It? **2. a.** What is the lateral area of the hexagonal pyramid at the right? Round to the nearest square foot.

b. **Reasoning** How does the slant height of a regular pyramid relate to its height? Explain.



Like a pyramid, a **cone** is a solid that has one base and a vertex that is not in the same plane as the base. However, the **base** of a cone is a circle. In a **right cone,** the **altitude** is a perpendicular segment from the **vertex** to the center of the base. The **height** h is the length of the altitude. The **slant height** ℓ is the distance from the vertex to a point on the edge of the base. In this book, you can assume that a cone is a right cone unless stated or pictured otherwise.



The **lateral area** is half the circumference of the base times the slant height. The formulas for the lateral area and **surface area** of a cone are similar to those for a pyramid.

take note

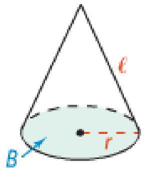
Theorem 11-4 Lateral and Surface Areas of a Cone

The lateral area of a right cone is half the product of the circumference of the base and the slant height of the cone.

$$\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot \ell, \text{ or L.A.} = \pi r \ell$$

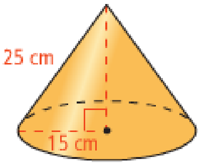
The surface area of a cone is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$



Problem 3 Finding the Surface Area of a Cone

What is the surface area of the cone in terms of π?



$\text{S.A.} = \text{L.A.} + B$ Use the formula for surface area.

$= \pi r\ell + \pi r^2$ Substitute the formulas for L.A. and B.

$= \pi(15)(25) + \pi(15)^2$ Substitute 15 for r and 25 for ℓ.

$= 375\pi + 225\pi$ Simplify.

$= 600\pi$

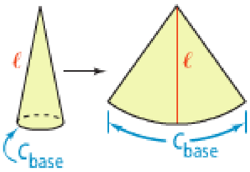
The surface area of the cone is 600π cm^2.

Think

How is this different from finding the surface area of a pyramid?
For a pyramid, you need to find the perimeter of the base. For a cone, you need to find the circumference.

Got It? **3.** The radius of the base of a cone is 16 m. Its slant height is 28 m. What is the surface area in terms of π?

By cutting a cone and laying it out flat, you can see how the formula for the lateral area of a cone $(\text{L.A.} = \frac{1}{2} \cdot C_{\text{base}} \cdot \ell)$ resembles that for the area of a triangle $(A = \frac{1}{2}bh)$.

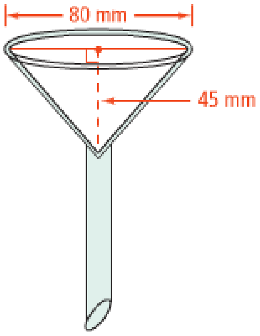


Think

What is the problem asking you to find?
The problem is asking you to find the area that the filter paper covers. This is the lateral area of a cone.

Problem 4 Finding the Lateral Area of a Cone

Chemistry **In a chemistry lab experiment, you use the conical filter funnel shown at the right. How much filter paper do you need to line the funnel?**



The top part of the funnel has the shape of a cone with a diameter of 80 mm and a height of 45 mm.

$\text{L.A.} = \pi r \ell$ — Use the formula for lateral area of a cone.

$= \pi r\left(\sqrt{r^2 + h^2}\right)$ — To find the slant height, use the Pythagorean Theorem.

$= \pi(40)\left(\sqrt{40^2 + 45^2}\right)$ — Substitute $\frac{1}{2} \cdot 80$, or 40, for r and 45 for h.

≈ 7565.957013 — Use a calculator.

You need about 7566 mm^2 of filter paper to line the funnel.

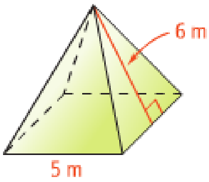
Got It? **4. a.** What is the lateral area of a traffic cone with radius 10 in. and height 28 in.? Round to the nearest whole number.

b. **Reasoning** Suppose the radius of a cone is halved, but the slant height remains the same. How does this affect the lateral area of the cone? Explain.

Lesson Check

Do you know HOW?

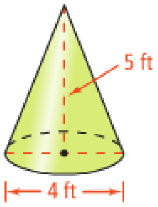
Use the diagram of the square pyramid at the right.



1. What is the lateral area of the pyramid?

2. What is the surface area of the pyramid?

Use the diagram of the cone at the right.



3. What is the lateral area of the cone?

4. What is the surface area of the cone?

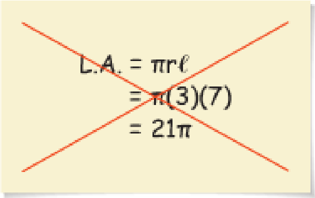
Do you UNDERSTAND?

5. **Vocabulary** How do the height and the slant height of a pyramid differ?

6. **Compare and Contrast** How are the formulas for the surface area of a prism and the surface area of a pyramid alike? How are they different?

7. **Vocabulary** How many lateral faces does a pyramid have if its base is pentagonal? Hexagonal? n-sided?

8. **Error Analysis** A cone has height 7 and radius 3. Your classmate calculates its lateral area. What is your classmate's error? Explain.



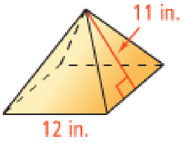
~~$\text{L.A.} = \pi r \ell$~~
~~$= \pi(3)(7)$~~
~~$= 21\pi$~~

Practice and Problem-Solving Exercises

A Practice

Find the surface area of each pyramid to the nearest whole number. **See Problem 1.**

9.



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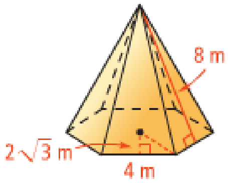

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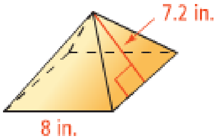
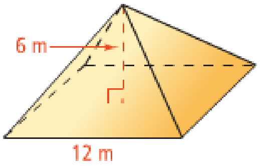


Find the lateral area of each pyramid to the nearest whole number. **See Problem 2.**

12.



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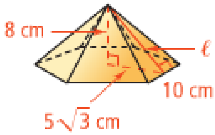

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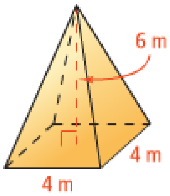
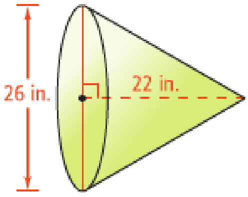


15. **Social Studies** The original height of the Pyramid of Khafre, located next to the Great Pyramid in Egypt, was about 471 ft. Each side of its square base was about 708 ft. What is the lateral area, to the nearest square foot, of a pyramid with those dimensions?

Find the lateral area of each cone to the nearest whole number. **See Problems 3 and 4.**

16.



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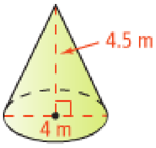

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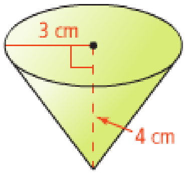
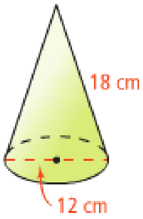


Find the surface area of each cone in terms of π.

19.



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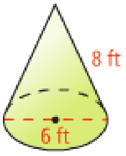

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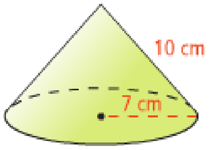
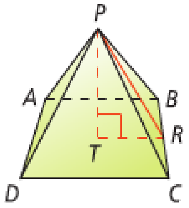


B Apply

22. **Reasoning** Suppose you could climb to the top of the Great Pyramid. Which route would be shorter, a route along a lateral edge or a route along the slant height of a side? Which of these routes is steeper? Explain your answers.

23. The lateral area of a cone is 4.8π in.^{2}. The radius is 1.2 in. Find the slant height.



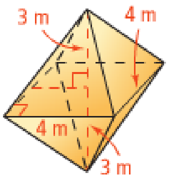
24. **Writing** Explain why the altitude $\overline{PT}$ in the pyramid at the right must be shorter than all of the lateral edges $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$.

25. **Think About a Plan** The lateral area of a pyramid with a square base is 240 ft^2. Its base edges are 12 ft long. Find the height of the pyramid.

- What additional information do you know about the pyramid based on the given information?
- How can a diagram help you identify what you need to find?

Find the surface area to the nearest whole number.

26.



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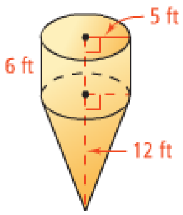

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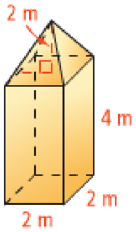
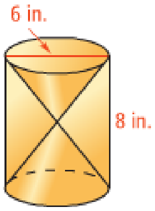


29. **Open-Ended** Draw a square pyramid with a lateral area of 48 cm^2. Label its dimensions. Then find its surface area.

30. **Architecture** The roof of a tower in a castle is shaped like a cone. The height of the roof is 30 ft and the radius of the base is 15 ft. What is the lateral area of the roof? Round your answer to the nearest tenth.



31. **Reasoning** The figure at the right shows two glass cones inside a cylinder. Which has a greater surface area, the two cones or the cylinder? Explain.

32. **Writing** You can use the formula S.A. $= (\ell + r)r\pi$ to find the surface area of a cone. Explain why this formula works. Also, explain why you may prefer to use this formula when finding surface area with a calculator.

33. Find a formula for each of the following.

a. the slant height of a cone in terms of the surface area and radius

b. the radius of a cone in terms of the surface area and slant height

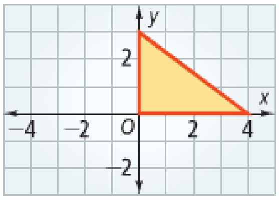
The length of a side (s) of the base, slant height (ℓ), height (h), lateral area (L.A.), and surface area (S.A.) are measurements of a square pyramid. Given two of the measurements, find the other three to the nearest tenth.

34. $s = 3$ in., S.A. $= 39$ $in.^2$

35. $h = 8$ m, $\ell = 10$ m

36. L.A. $= 118$ cm^2, S.A. $= 182$ cm^2

Visualization **Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid. Then find its surface area in terms of π.**



37. the y-axis

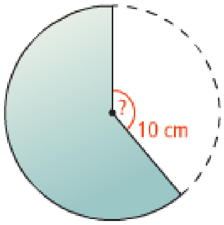
38. the x-axis

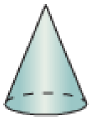
Challenge

39. the line $x = 4$

40. the line $y = 3$

41. A sector has been cut out of the disk at the right. The radii of the part that remains are taped together, without overlapping, to form the cone. The cone has a lateral area of 64π cm^2. Find the measure of the central angle of the cutout sector.





Each given figure fits inside a 10-cm cube. The figure's base is in one face of the cube and is as large as possible. The figure's vertex is in the opposite face of the cube. Draw a sketch and find the lateral and surface areas of the figure.

42. a square pyramid

43. a cone

Standardized Test Prep

SAT/ACT

44. To the nearest whole number, what is the surface area of a cone with diameter 27 m and slant height 19 m?

Ⓐ 1378 m^2 Ⓑ 1951 m^2 Ⓒ 2757 m^2 Ⓓ 3902 m^2

45. What is the hypotenuse of a right isosceles triangle with leg $2\sqrt{6}$?

Ⓕ $4\sqrt{6}$ Ⓖ $2\sqrt{3}$ Ⓗ $4\sqrt{3}$ Ⓘ $4\sqrt{2}$

46. Two angles in a triangle have measures 54 and 61. What is the measure of the smallest exterior angle?

Ⓐ 119 Ⓑ 115 Ⓒ 112 Ⓓ 126

Short Response

47. A diagonal divides a parallelogram into two isosceles triangles. Can the parallelogram be a rhombus? Explain.

Extended Response

48. $ABCD$ has vertices at $A(3, 4)$, $B(7, 5)$, $C(6, 1)$, and $D(-2, -4)$. A dilation with center $(0, 0)$ maps A to $A'\left(\frac{15}{2}, 10\right)$. What are the coordinates of B', C', and D'? Show your work.

Mixed Review

49. How much cardboard do you need to make a closed box 4 ft by 5 ft by 2 ft? **See Lesson 11-2.**

50. How much posterboard do you need to make a cylinder, open at each end, with height 9 in. and diameter $4\frac{1}{2}$ in.? Round your answer to the nearest square inch.

51. A kite with area 195 in.^{2} has a 15-in. diagonal. How long is the other diagonal? **See Lesson 10-2.**

Get Ready! **To prepare for Lesson 11-4, do Exercises 52 and 53.**

Find the area of each figure. If necessary, round to the nearest tenth. **See Lessons 1-8 and 10-7.**

52. a square with side length 2 cm

53. a circle with diameter 15 in.

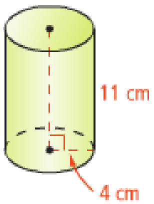
11 Mid-Chapter Quiz

MathXL® for School
Go to PowerGeometry.com

Do you know HOW?

Draw a net for each figure. Label the net with its dimensions.

1.



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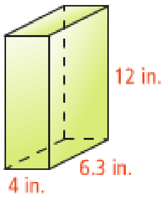

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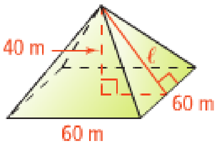

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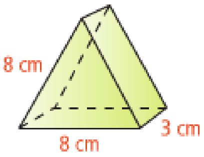


Find the surface area of each solid. Round to the nearest square unit.

5. cylinder in Exercise 1

6. prism in Exercise 2

7. pyramid in Exercise 3

8. cone in Exercise 4

For Exercises 9–11, use Euler's Formula. Show your work.

9. A polyhedron has 10 vertices and 15 edges. How many faces does it have?

10. A polyhedron has 2 hexagonal faces and 12 triangular faces. How many vertices does it have?

11. How many vertices does the net of a pentagonal pyramid have?

Draw a cube. Shade a cross section of the cube that forms each shape.

12. a rectangle

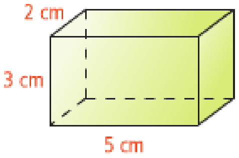
13. a trapezoid

14. a triangle

15. A square prism with base edges 2 in. has surface area 32 in.^{2}. What is its height?

Find the surface area of each figure to the nearest whole number.

16.



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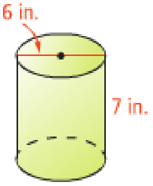

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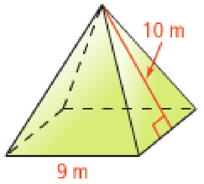

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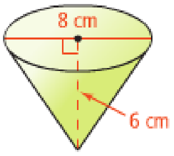


Do you UNDERSTAND?

20. **Open-Ended** Draw a net for a regular hexagonal prism.

21. **Compare and Contrast** How are a right prism and a regular pyramid alike? How are they different?

22. **Algebra** The height of a cylinder is twice the radius of the base. A cube has an edge length equal to the radius of the cylinder. Find the ratio between the surface area of the cylinder and the surface area of the cube. Leave your answer in terms of π.

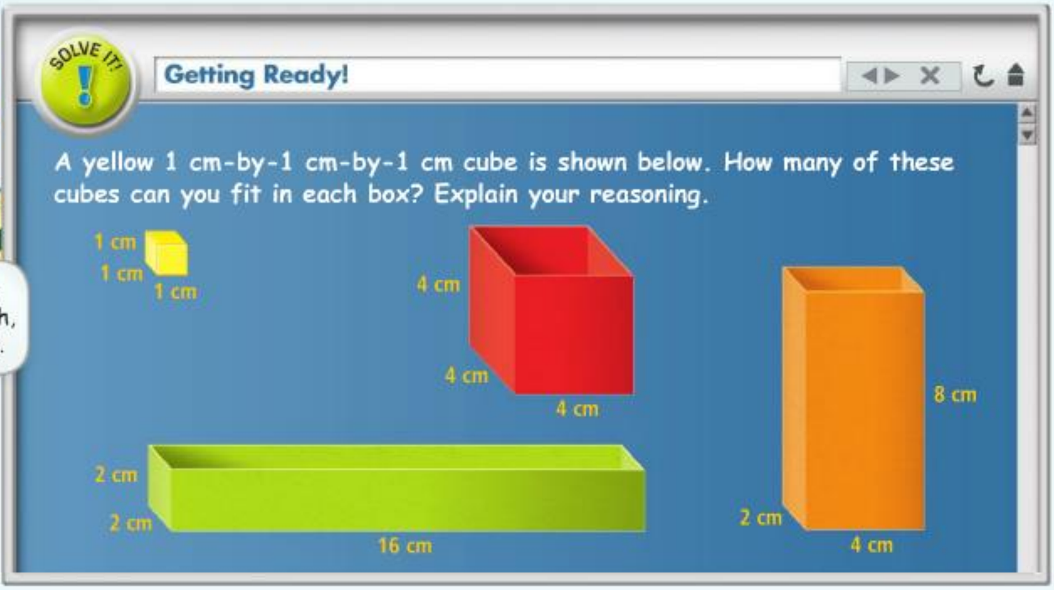
23. **Error Analysis** Your class learned that the number of regions in the net for a solid is the same as the number of faces in that solid. Your friend concludes that a solid and its net must also have the same number of edges. What is your friend's error? Explain.

24. **Visualization** The dimensions of a rectangular prism are 3 in. by 5 in. by 10 in. Is it possible to intersect this prism with a plane so that the resulting cross section is a square? If so, draw and describe your cross section. Indicate the length of the edge of the square.

11-4 Volumes of Prisms and Cylinders

Objective To find the volume of a prism and the volume of a cylinder

A cube has three dimensions: length, width, and height.



Getting Ready!

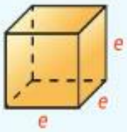
A yellow 1 cm-by-1 cm-by-1 cm cube is shown below. How many of these cubes can you fit in each box? Explain your reasoning.

Dynamic Activity
Volumes of Prisms and Cylinders

Lesson Vocabulary
- volume
- composite space figure

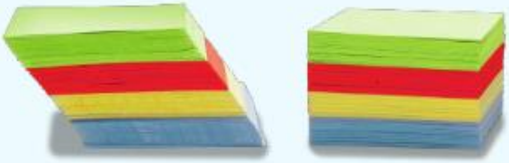
In the Solve It, you determined the volume of a box by finding how many 1 cm-by-1 cm-by-1 cm cubes the box holds.

Volume is the space that a figure occupies. It is measured in cubic units such as cubic inches (in.^{3}), cubic feet (ft^3), or cubic centimeters (cm^3). The volume V of a cube is the cube of the length of its edge e, or $V = e^3$.



Essential Understanding You can find the volume of a prism or a cylinder when you know its height and the area of its base.

Both stacks of paper below contain the same number of sheets.



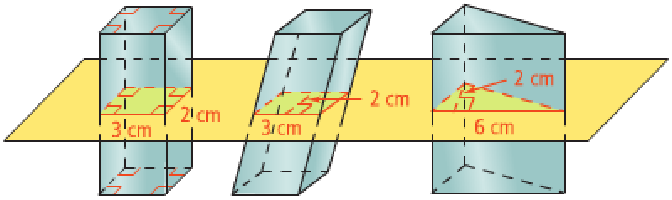
The first stack forms an oblique prism. The second forms a right prism. The stacks have the same height. The area of every cross section parallel to a base is the area of one sheet of paper. The stacks have the same volume. These stacks illustrate the following principle.

take note

Theorem 11-5 Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.

The area of each shaded cross section below is 6 cm^2. Since the prisms have the same height, their volumes must be the same by Cavalieri's Principle.



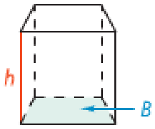
You can find the volume of a right prism by multiplying the area of the base by the height. Cavalieri's Principle lets you extend this idea to any prism.

take note

Theorem 11-6 Volume of a Prism

The volume of a prism is the product of the area of the base and the height of the prism.

$$V = Bh$$



Problem 1 Finding the Volume of a Rectangular Prism

Plan

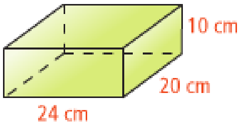
What do you need to use the formula?
You need to find B, the area of the base. The prism has a rectangular base, so the area of the base is length × width.

What is the volume of the rectangular prism at the right?

$V = Bh$ Use the formula for the volume of a prism.

$= 480 \cdot 10$ The area of the base B is $24 \cdot 20$, or 480 cm^2, and the height is 10 cm.

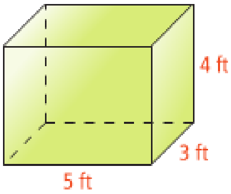
$= 4800$ Simplify.



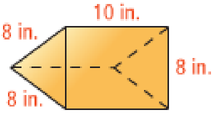
The volume of the rectangular prism is 4800 cm^3.

Got It? **1. a.** What is the volume of the rectangular prism at the right?

b. Reasoning Suppose the prism at the right is turned so that the base is 4 ft by 5 ft and the height is 3 ft. Does the volume change? Explain.



Problem 2 Finding the Volume of a Triangular Prism

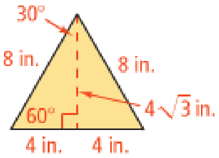


Multiple Choice What is the approximate volume of the triangular prism?

Ⓐ 188 in.^{3} Ⓒ 295 in.^{3}

Ⓑ 277 in.^{3} Ⓓ 554 in.^{3}

Step 1 Find the area of the base of the prism.



Each base of the triangular prism is an equilateral triangle, as shown at the right. An altitude of the triangle divides it into two 30°-60°-90° triangles. The height of the triangle is $\sqrt{3}$ · shorter leg, or $4\sqrt{3}$.

$B = \frac{1}{2}bh$ Use the formula for the area of a triangle.

$= \frac{1}{2}(8)(4\sqrt{3})$ Substitute 8 for *b* and $4\sqrt{3}$ for *h*.

$= 16\sqrt{3}$ Simplify.

Think

Which height do you use in the formula? Remember that the *h* in the formula for volume represents the height of the entire prism, not the height of the triangular base.

Step 2 Find the volume of the prism.

$V = Bh$ Use the formula for the volume of a prism.

$= 16\sqrt{3} \cdot 10$ Substitute $16\sqrt{3}$ for *B* and 10 for *h*.

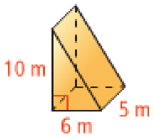
$= 160\sqrt{3}$ Simplify.

≈ 277.1281292 Use a calculator.

The volume of the triangular prism is about 277 in.^{3}. The correct answer is B.

Got It? **2. a.** What is the volume of the triangular prism at the right?

b. **Reasoning** Suppose the height of a prism is doubled. How does this affect the volume of the prism? Explain.



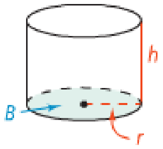
To find the volume of a cylinder, you use the same formula $V = Bh$ that you use to find the volume of a prism. Now, however, B is the area of the circle, so you use the formula $B = \pi r^2$ to find its value.

take note

Theorem 11-7 Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.

$$V = Bh, \text{ or } V = \pi r^2 h$$

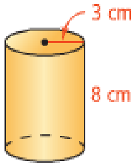


Plan

What do you know from the diagram?
You know that the radius r is 3 cm and the height h is 8 cm.

Problem 3 Finding the Volume of a Cylinder

What is the volume of the cylinder in terms of π?

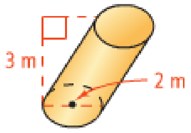


$$\begin{aligned} V &= \pi r^2 h && \text{Use the formula for the volume of a cylinder.} \\ &= \pi(3)^2(8) && \text{Substitute 3 for } r \text{ and 8 for } h. \\ &= \pi(72) && \text{Simplify.} \end{aligned}$$

The volume of the cylinder is 72π cm^3.

Got It? **3. a.** What is the volume of the cylinder at the right in terms of π?

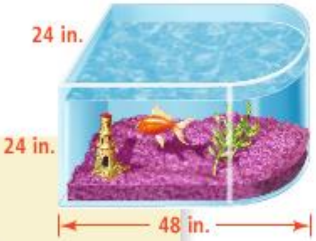
b. Reasoning Suppose the radius of a cylinder is halved. How does this affect the volume of the cylinder? Explain.



A **composite space figure** is a three-dimensional figure that is the combination of two or more simpler figures. You can find the volume of a composite space figure by adding the volumes of the figures that are combined.

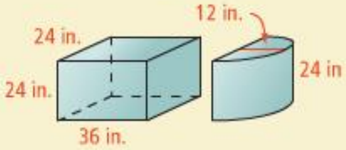
Problem 4 Finding Volume of a Composite Figure

What is the approximate volume of the bullnose aquarium to the nearest cubic inch?

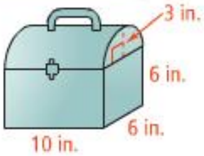

24 in.
24 in.
48 in.

Plan

How can you find the volume by *solving a simpler problem*?
The aquarium is the combination of a rectangular prism and half of a cylinder. Find the volume of each figure.

Think	Write
The length of the prism is the total length minus the radius of the cylinder. The radius of the cylinder is half the width of the prism.	
Find the volume of the prism and the half cylinder.	$V_1 = Bh$ $= (24 \cdot 36)(24)$ $= 20{,}736$ $V_2 = \frac{1}{2}\pi r^2 h$ $= \frac{1}{2}\pi(12)^2(24)$ ≈ 5429
Add the two volumes together.	$20{,}736 + 5429 = 26{,}165$ The approximate volume of the aquarium is 26,165 in.^{3}.

Got It? **4.** What is the approximate volume of the lunch box shown at the right? Round to the nearest cubic inch.

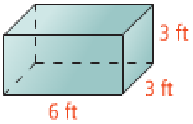


Lesson Check

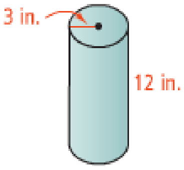
Do you know HOW?

What is the volume of each figure? If necessary, round to the nearest whole number.

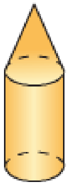
1.



2.



Do you UNDERSTAND?



3. Vocabulary Is the figure at the right a composite space figure? Explain.

4. Compare and Contrast How are the formulas for the volume of a prism and the volume of a cylinder alike? How are they different?

5. Reasoning How is the volume of a rectangular prism with base 2 m by 3 m and height 4 m related to the volume of a rectangular prism with base 3 m by 4 m and height 2 m? Explain.

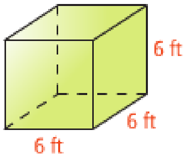
Practice and Problem-Solving Exercises

A Practice

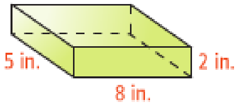
Find the volume of each rectangular prism.

See Problem 1.

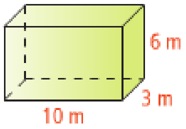
6.



7.



8.

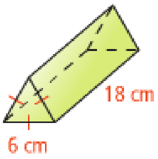


9. The base is a square with sides of 2 cm. The height is 3.5 cm.

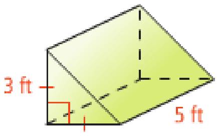
Find the volume of each triangular prism.

See Problem 2.

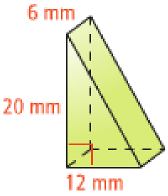
10.



11.



12.

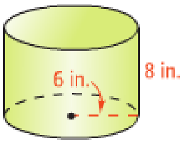


13. The base is a 45°-45°-90° triangle with a leg of 5 in. The height is 1.8 in.

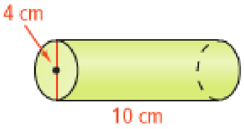
Find the volume of each cylinder in terms of π and to the nearest tenth.

See Problem 3.

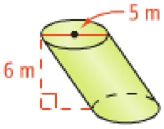
14.



15.



16.



17. The diameter of the cylinder is 1 yd. The height is 4 yd.

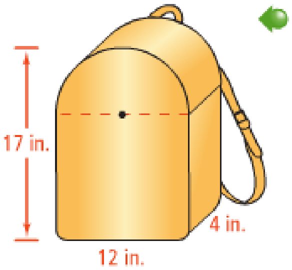
See Problem 4.

18. **Composite Figures** Use the diagram of the backpack at the right.

a. What two figures approximate the shape of the backpack?

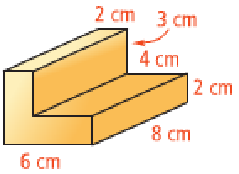
b. What is the volume of the backpack in terms of π?

c. What is the volume of the backpack to the nearest cubic inch?

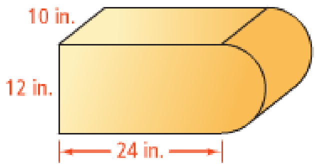


Find the volume of each composite space figure to the nearest whole number.

19.



20.



B Apply

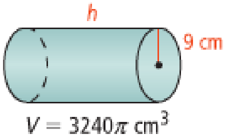
21. **Think About a Plan** A full waterbed mattress is 7 ft by 4 ft by 1 ft. If water weighs 62.4 lb/ft^3, what is the weight of the water in the mattress to the nearest pound?

- How can you determine the amount of water the mattress can hold?
- The weight of the water is in pounds per cubic feet. How can you get an answer with a unit of pounds?

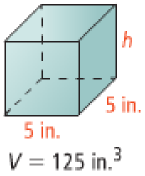
22. **Open-Ended** Give the dimensions of two rectangular prisms that have volumes of 80 cm^3 each but also have different surface areas.

Find the height of each figure with the given volume.

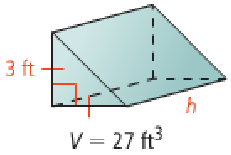
23.



24.

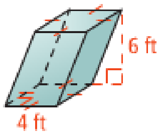


25.



26. **Sports** A can of tennis balls has a diameter of 3 in. and a height of 8 in. Find the volume of the can to the nearest cubic inch.

27. What is the volume of the oblique prism shown at the right?



28. **Environmental Engineering** A scientist suggests keeping indoor air relatively clean as follows: For a room with a ceiling 8 ft high, provide two or three pots of flowers for every 100 ft^2 of floor space. If your classroom has an 8-ft ceiling and measures 35 ft by 40 ft, how many pots of flowers should it have?

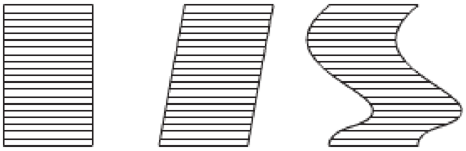
29. **Reasoning** Suppose the dimensions of a prism are tripled. How does this affect its volume? Explain.

30. **Swimming Pool** The approximate dimensions of an Olympic-size swimming pool are 164 ft by 82 ft by 6.6 ft.

a. Find the volume of the pool to the nearest cubic foot.

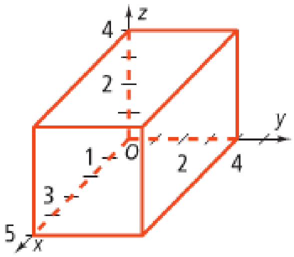
b. If 1 $\text{ft}^3 \approx 7.48$ gal, about how many gallons does the pool hold?

31. **Writing** The figures at the right can be covered by equal numbers of straws that are the same length. Describe how Cavalieri's Principle could be adapted to compare the areas of these figures.



32. **Algebra** The volume of a cylinder is 600π cm^3. The radius of a base of the cylinder is 5 cm. What is the height of the cylinder?

33. **Coordinate Geometry** Find the volume of the rectangular prism at the right.



34. **Algebra** The volume of a cylinder is 135π cm^3. The height of the cylinder is 15 cm. What is the radius of a base of the cylinder?

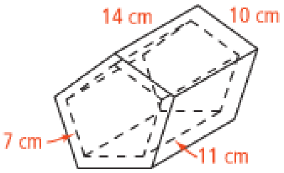
35. **Landscaping** To landscape her 70 ft-by-60 ft rectangular backyard, your aunt is planning first to put down a 4-in. layer of topsoil. She can buy bags of topsoil at \$2.50 per 3-ft^3 bag, with free delivery. Or, she can buy bulk topsoil for \$22.00/yd^3, plus a \$20 delivery fee. Which option is less expensive? Explain.

36. The closed box at the right is shaped like a regular pentagonal prism. The exterior of the box has base edge 10 cm and height 14 cm. The interior has base edge 7 cm and height 11 cm. Find each measurement.

a. the outside surface area

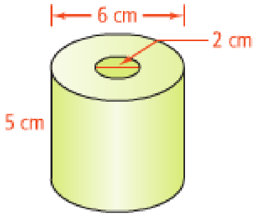
b. the inside surface area

c. the volume of the material needed to make the box

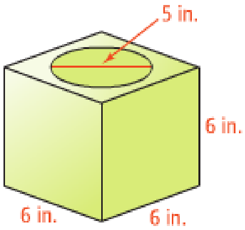


A cylinder has been cut out of each solid. Find the volume of the remaining solid. Round your answer to the nearest tenth.

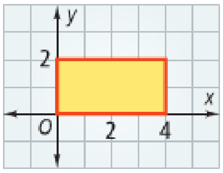
37.



38.



Visualization **Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid and find its volume in terms of π.**



39. the x-axis

40. the y-axis

41. the line $y = 2$

42. the line $x = 5$

Challenge

43. **Paper Folding** Any rectangular sheet of paper can be rolled into a right cylinder in two ways.
   - a. Use ordinary sheets of paper to model the two cylinders. Compute the volume of each cylinder. How do they compare?
   - b. Of all sheets of paper with perimeter 39 in., which size can be rolled into a right cylinder with greatest volume? (*Hint*: Try making a table.)

44. **Plumbing** The outside diameter of a pipe is 5 cm. The inside diameter is 4 cm. The pipe is 4 m long. What is the volume of the material used for this length of pipe? Round your answer to the nearest cubic centimeter.

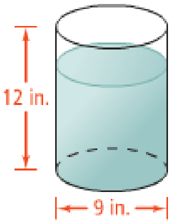
45. The radius of Cylinder B is twice the radius of Cylinder A. The height of Cylinder B is half the height of Cylinder A. Compare their volumes.

Standardized Test Prep

SAT/ACT

46. The water surface is 2.5 in. from the top of the cylindrical water tank at the right. About how much water is in the tank?

Ⓐ 604 in.^{3} Ⓑ 636 in.^{3} Ⓒ 668 in.^{3} Ⓓ 763 in.^{3}



47. Which statement is logically equivalent to the following statement?
If a figure is a rhombus, then it has four sides.

Ⓕ If a figure is not a rhombus, then it does not have four sides.
Ⓖ If a figure is a rectangle, then it has four sides.
Ⓗ If a figure does not have four sides, then it is not a rhombus.
Ⓘ If a figure has four sides, then it could be a rhombus.

48. Which of the following is NOT an undefined term?

Ⓐ point Ⓑ line Ⓒ ray Ⓓ plane

Short Response

49. How is the formula for finding the lateral area of a cylinder similar to the formula for finding the area of a rectangle?

Mixed Review

Find the lateral area of each figure to the nearest tenth. **See Lesson 11-3.**

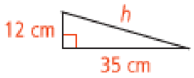
50. a right circular cone with height 12 mm and radius 5 mm

51. a regular hexagonal pyramid with base edges 9.2 ft long and slant height 17 ft

Get Ready! **To prepare for Lesson 11-5, do Exercises 52 and 53.**

See Lesson 8-1.

52. Find h in the figure at the right.



53. A right triangle has hypotenuse 300 ft and leg 180 ft. What is the length of the other leg?

Concept Byte

Use With Lesson 11-5

ACTIVITY

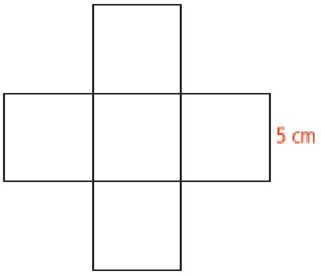
Finding Volume

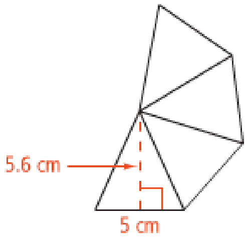
You know how to find the volumes of a prism and of a cylinder. Use the following activities to explore finding the volumes of a pyramid and of a cone.

Activity 1

Step 1 Draw the nets shown at the right on heavy paper.

Step 2 Cut out the nets and tape them together to make a cube and a square pyramid. Each model will have one open face.



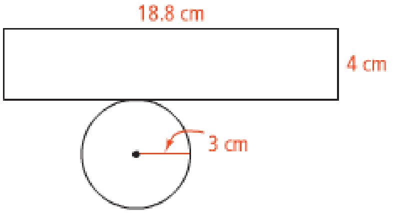


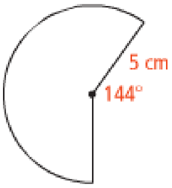
1. How do the areas of the bases of the cube and the pyramid compare?
2. How do the heights of the cube and the pyramid compare?
3. Fill the pyramid with rice or other material. Then pour the rice from the pyramid into the cube. How many pyramids full of rice does the cube hold?
4. The volume of the pyramid is what fractional part of the volume of the cube?
5. **Make a Conjecture** What do you think is the formula for the volume of a pyramid? Explain.

Activity 2

Step 1 Draw the nets shown at the right on heavy paper.

Step 2 Cut out the nets and tape them together to make a cylinder and a cone. Each model will have one open face.



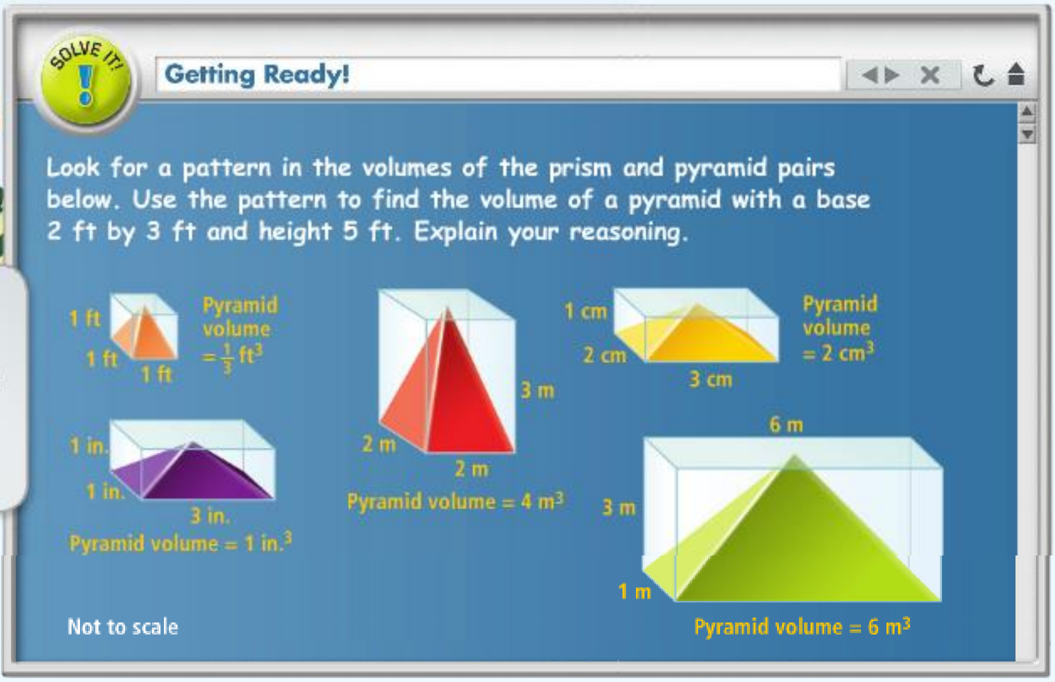


6. How do the areas of the bases of the cylinder and of the cone compare?
7. How do the heights of the cylinder and of the cone compare?
8. Fill the cone with rice or other material. Then pour the rice from the cone into the cylinder. How many cones full of rice does the cylinder hold?
9. What fractional part of the volume of the cylinder is the volume of the cone?
10. **Make a Conjecture** What do you think is the formula for the volume of a cone? Explain.

11-5 Volumes of Pyramids and Cones

Objective To find the volume of a pyramid and of a cone

There is a relationship between the volume of a prism and the volume of a pyramid embedded in the prism.



Getting Ready!

Look for a pattern in the volumes of the prism and pyramid pairs below. Use the pattern to find the volume of a pyramid with a base 2 ft by 3 ft and height 5 ft. Explain your reasoning.

Dynamic Activity
Volumes of Pyramids and Cones

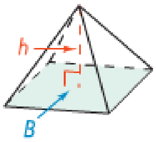
In the Solve It, you analyzed the relationship between the volume of a prism and the volume of an embedded pyramid.

Essential Understanding The volume of a pyramid is related to the volume of a prism with the same base and height.

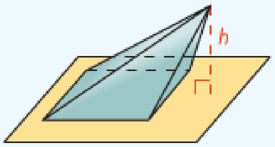
take note Theorem 11-8 Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

$$V = \frac{1}{3}Bh$$



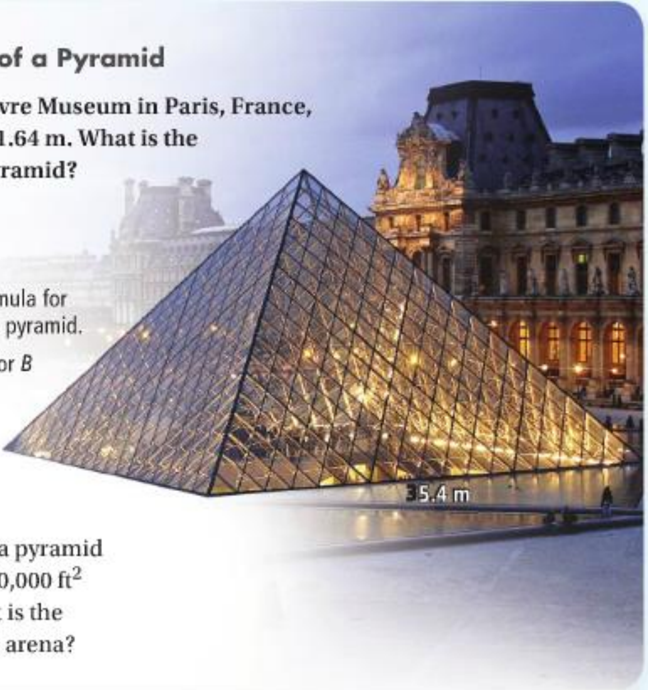
Because of Cavalieri's Principle, the volume formula is true for all pyramids. The height *h* of an oblique pyramid is the length of the perpendicular segment from its vertex to the plane of the base.



Oblique pyramid

Problem 1 Finding Volume of a Pyramid

Architecture **The entrance to the Louvre Museum in Paris, France, is a square pyramid with a height of 21.64 m. What is the approximate volume of the Louvre Pyramid?**



Think

How is this similar to finding the volume of a prism?
In both cases, you need the area of the base and the height.

The area of the base of the pyramid is 35.4 m · 35.4 m, or 1253.16 m^2.

$V = \frac{1}{3}Bh$	Use the formula for volume of a pyramid.
$= \frac{1}{3}(1253.16)(21.64)$	Substitute for B and h.
$= 9039.4608$	Simplify.

The volume is about 9039 m^3.

Got It? **1.** A sports arena shaped like a pyramid has a base area of about 300,000 ft^2 and a height of 321 ft. What is the approximate volume of the arena?

Problem 2 Finding the Volume of a Pyramid

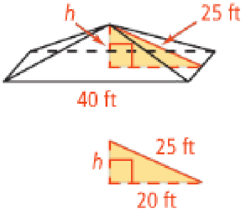
GRIDDED RESPONSE

What is the volume in cubic feet of a square pyramid with base edges 40 ft and slant height 25 ft?

Think

How do you use the slant height?
The slant height is the length of the hypotenuse of the right triangle. Use the slant height to find the height of the pyramid.

Step 1 Find the height of the pyramid.

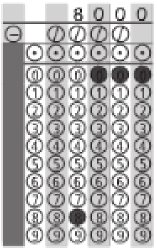


$c^2 = a^2 + b^2$	Use the Pythagorean Theorem.
$25^2 = h^2 + 20^2$	Substitute 25 for c, h for a, and $\frac{40}{2}$, or 20, for b.
$625 = h^2 + 400$	Simplify.
$h^2 = 225$	Solve for h^2.
$h = 15$	Take the positive square root of both sides.

Step 2 Find the volume of the pyramid.

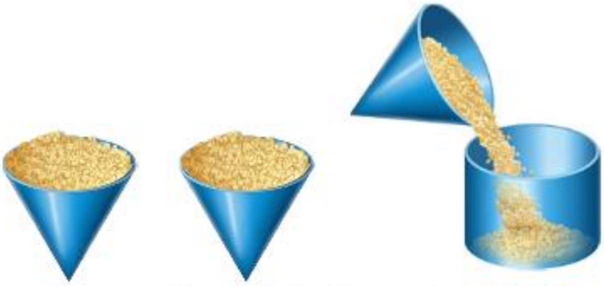
$V = \frac{1}{3}Bh$	Use the formula for volume of a pyramid.
$= \frac{1}{3}(40 \cdot 40)(15)$	Substitute 40 · 40 for B and 15 for h.
$= 8000$	Simplify.

The volume of the pyramid is 8000 ft^3.



Got It? **2.** What is the volume of a square pyramid with base edges 24 m and slant height 13 m?

Essential Understanding The volume of a cone is related to the volume of a cylinder with the same base and height.



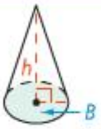
The cones and the cylinder have the same base and height.
It takes three cones full of rice to fill the cylinder.

take note

Theorem 11-9 Volume of a Cone

The volume of a cone is one third the product of the area of the base and the height of the cone.

$$V = \frac{1}{3}Bh, \text{ or } V = \frac{1}{3}\pi r^2 h$$



A cone-shaped structure can be particularly strong, as downward forces at the vertex are distributed to all points in its circular base.

Problem 3 Finding the Volume of a Cone

Think

How is this similar to finding the volume of a cylinder?
In both cases, you need to find the base area of a circle.

Traditional Architecture The covering on a tepee rests on poles that come together like concurrent lines. The resulting structure approximates a cone. If the tepee pictured is 12 ft high with a base diameter of 14 ft, what is its approximate volume?

$V = \frac{1}{3}\pi r^2 h$ Use the formula for the volume of a cone.

$= \frac{1}{3}\pi(7)^2(12)$ Substitute $\frac{14}{2}$, or 7, for r and 12 for h.

≈ 615.7521601 Use a calculator.

The volume of the tepee is approximately 616 ft^3.

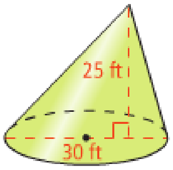
Got It? **3. a.** The height and radius of a child's tepee are half those of the tepee in Problem 3. What is the volume of the child's tepee to the nearest cubic foot?

b. Reasoning What is the relationship between the volume of the original tepee and the child's tepee?

This volume formula applies to all cones, including oblique cones.

Problem 4 Finding the Volume of an Oblique Cone

What is the volume of the oblique cone at the right? Give your answer in terms of π and also rounded to the nearest cubic foot.



$V = \frac{1}{3}\pi r^2 h$ Use the formula for volume of a cone.

$= \frac{1}{3}\pi(15)^2(25)$ Substitute 15 for r and 25 for h.

$= 1875\pi$ Simplify.

≈ 5890.486225 Use a calculator.

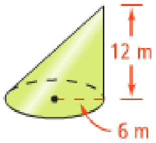
The volume of the cone is 1875π ft^3, or about 5890 ft^3.

Think

What is the height of the oblique cone?
The height is the length of the perpendicular segment from the vertex of the cone to the base, which is 25 ft. In an oblique cone, the segment does not intersect the center of the base.

Got It? **4. a.** What is the volume of the oblique cone at the right in terms of π and rounded to the nearest cubic meter?

b. Reasoning How does the volume of an oblique cone compare to the volume of a right cone with the same diameter and height? Explain.

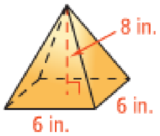


Lesson Check

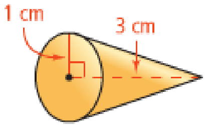
Do you know HOW?

What is the volume of each figure? If necessary, round to the nearest tenth.

1.



2.



Do you UNDERSTAND?

3. Compare and Contrast How are the formulas for the volume of a pyramid and the volume of a cone alike? How are they different?

4. Error Analysis A square pyramid has base edges 13 ft and height 10 ft. A cone has diameter 13 ft and height 10 ft. Your friend claims the figures have the same volume because the volume formulas for a pyramid and a cone are the same: $V = \frac{1}{3}Bh$. What is her error?

Practice and Problem-Solving Exercises

A Practice

Find the volume of each square pyramid.

See Problem 1.

5. base edges 10 cm, height 6 cm

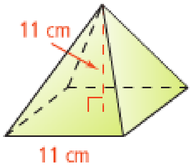
6. base edges 18 in., height 12 in.

7. base edges 5 m, height 6 m

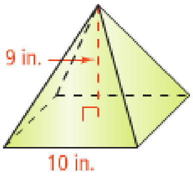
8. Buildings The Transamerica Pyramid Building in San Francisco is 853 ft tall with a square base that is 149 ft on each side. To the nearest thousand cubic feet, what is the volume of the Transamerica Pyramid?

Find the volume of each square pyramid. Round to the nearest tenth if necessary.

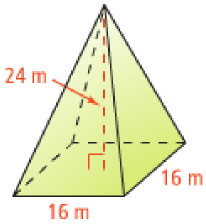
9.



10.



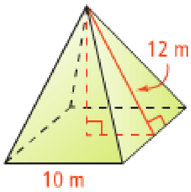
11.



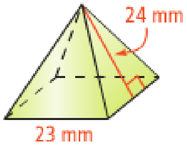
Find the volume of each square pyramid, given its slant height. Round to the nearest tenth.

See Problem 2.

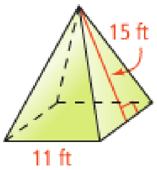
12.



13.



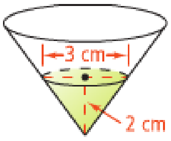
14.



15. **Chemistry** In a chemistry lab you use a filter paper cone to filter a liquid. The diameter of the cone is 6.5 cm and its height is 6 cm. How much liquid will the cone hold when it is full?

See Problem 3.

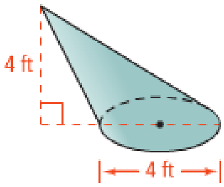
16. **Chemistry** This cone has a filter that was being used to remove impurities from a solution but became clogged and stopped draining. The remaining solution is represented by the shaded region. How many cubic centimeters of the solution remain in the cone?



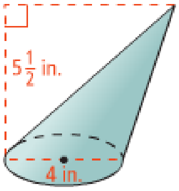
Find the volume of each cone in terms of π and also rounded as indicated.

See Problem 4.

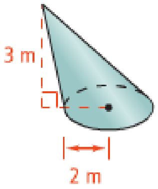
17. nearest cubic foot



18. nearest cubic inch



19. nearest cubic meter



B Apply

20. **Think About a Plan** A cone with radius 1 fits snugly inside a square pyramid, which fits snugly inside a cube. What are the volumes of the three figures?

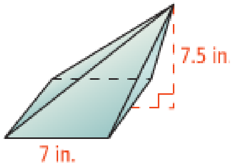
- How can you *draw a diagram* of the situation?
- What dimensions do the cone, pyramid, and cube have in common?

21. **Reasoning** Suppose the height of a pyramid is halved. How does this affect its volume? Explain.

22. **Writing** Without doing any calculations, explain how the volume of a cylinder with $B = 5\pi \text{ cm}^2$ and $h = 20$ cm compares to the volume of a cone with the same base area and height.

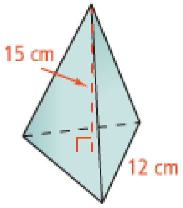
Find the volume to the nearest whole number.

23.



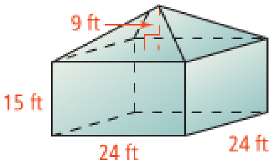
Square base

24.



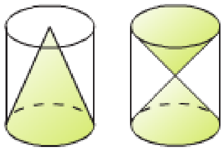
Equilateral base

25.



Square base

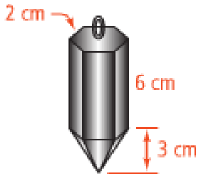
26. **Writing** The two cylinders pictured at the right are congruent. How does the volume of the larger cone compare to the total volume of the two smaller cones? Explain.



27. **Architecture** The Pyramid of Peace is an opera house in Astana, Kazakhstan. The height of the pyramid is approximately 62 m and one side of its square base is approximately 62 m.

- **a.** What is its volume to the nearest thousand cubic meters?
- **b.** How tall would a prism-shaped building with the same square base as the Pyramid of Peace have to be to have the same volume as the pyramid?

28. **Hardware** Builders use a plumb bob to find a vertical line. The plumb bob shown at the right combines a regular hexagonal prism with a pyramid. Find its volume to the nearest cubic centimeter.

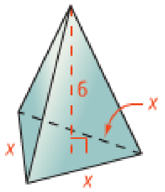


29. **Reasoning** A cone with radius 3 ft and height 10 ft has a volume of 30π ft^3. What is the volume of the cone formed when the following happens to the original cone?

- **a.** The radius is doubled.
- **b.** The height is doubled.
- **c.** The radius and the height are both doubled.

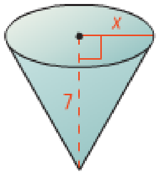
Algebra **Find the value of the variable in each figure. Leave answers in simplest radical form. The diagrams are not to scale.**

30.



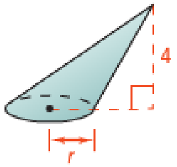
Volume $= 18\sqrt{3}$

31.



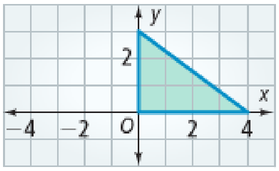
Volume $= 21\pi$

32.



Volume $= 24\pi$

Visualization **Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid. Then find its volume in terms of π.**



33. the y-axis

34. the x-axis

Challenge

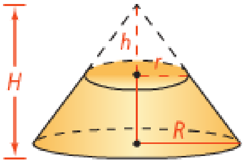
35. the line $x = 4$

36. the line $y = -1$

37. A *frustum* of a cone is the part that remains when the top of the cone is cut off by a plane parallel to the base.

a. Explain how to use the formula for the volume of a cone to find the volume of a frustum of a cone.

b. **Containers** A popcorn container 9 in. tall is the frustum of a cone. Its small radius is 4.5 in. and its large radius is 6 in. What is its volume?



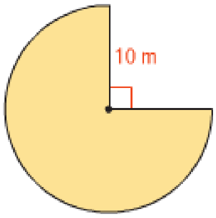
Frustum of cone

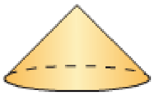
38. A disk has radius 10 m. A 90° sector is cut away, and a cone is formed.

a. What is the circumference of the base of the cone?

b. What is the area of the base of the cone?

c. What is the volume of the cone? (*Hint:* Use the slant height and the radius of the base to find the height.)





Standardized Test Prep

SAT/ACT

39. A cone has diameter 8 in. and height 14 in. A rectangular prism is 6 in. by 4 in. by 10 in. A square pyramid has base edge 8 in. and height 12 in. What are the volumes of the three figures in order from least to greatest?

Ⓐ cone, prism, pyramid
Ⓑ prism, cone, pyramid
Ⓒ pyramid, cone, prism
Ⓓ prism, pyramid, cone

40. One row of a truth table lists p as true and q as false. Which of the following statements is true?

Ⓕ $p \rightarrow q$
Ⓖ $p \vee q$
Ⓗ $p \wedge q$
Ⓘ $\sim p$

41. If a polyhedron has 8 vertices and 12 edges, how many faces does it have?

Ⓐ 4
Ⓑ 6
Ⓒ 12
Ⓓ 24

Extended Response

42. The point of concurrency of the three altitudes of a triangle lies outside the triangle. Where are its circumcenter, incenter, and centroid located in relation to the triangle? Draw diagrams to support your answers.

Mixed Review

43. A triangular prism has height 30 cm. Its base is a right triangle with legs 10 cm and 24 cm. What is the volume of the prism?

See Lesson 11-4.

44. Given $\triangle JAC$ and $\triangle KIN$, you know $\overline{JA} \cong \overline{KI}$, $\overline{AC} \cong \overline{IN}$, and $m\angle A > m\angle I$. What can you conclude about JC and KN?

See Lesson 5-7.

Get Ready! **To prepare for Lesson 11-6, do Exercises 45 and 46.**

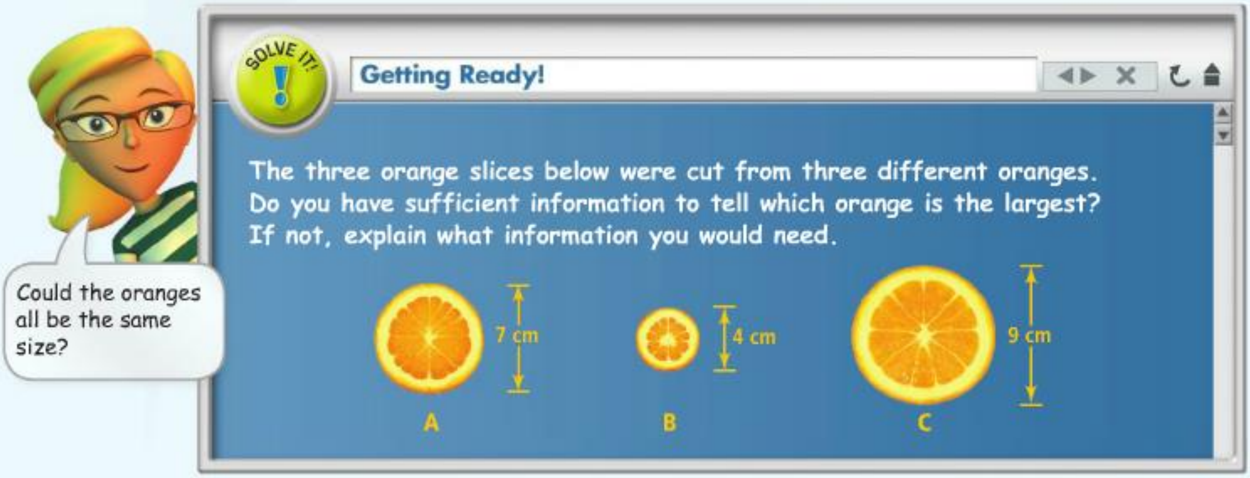
45. Find the area of a circle with diameter 3 in. to the nearest tenth.

See Lesson 1-8.

46. Find the circumference of a circle with radius 2 cm to the nearest centimeter.

11-6 Surface Areas and Volumes of Spheres

Objective To find the surface area and volume of a sphere



Could the oranges all be the same size?

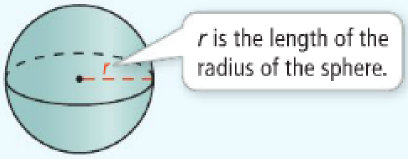
Getting Ready!

The three orange slices below were cut from three different oranges. Do you have sufficient information to tell which orange is the largest? If not, explain what information you would need.

Lesson Vocabulary
- sphere
- center of a sphere
- radius of a sphere
- diameter of a sphere
- circumference of a sphere
- great circle
- hemisphere

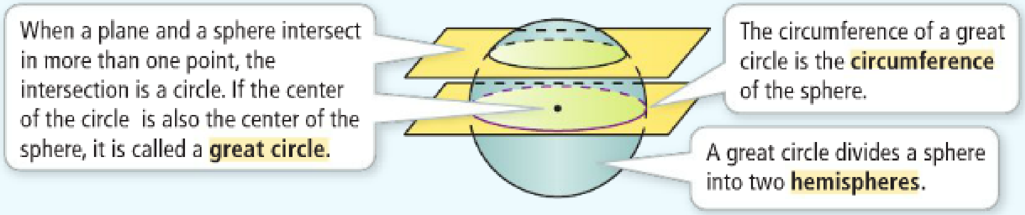
In the Solve It, you considered the sizes of objects with circular cross sections.

A **sphere** is the set of all points in space equidistant from a given point called the **center.** A **radius** is a segment that has one endpoint at the center and the other endpoint on the sphere. A **diameter** is a segment passing through the center with endpoints on the sphere.



r is the length of the radius of the sphere.

Essential Understanding You can find the surface area and the volume of a sphere when you know its radius.

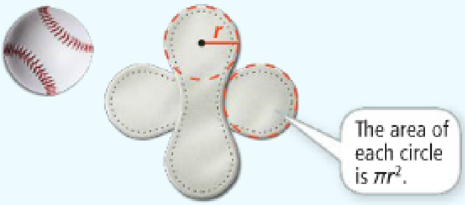


When a plane and a sphere intersect in more than one point, the intersection is a circle. If the center of the circle is also the center of the sphere, it is called a **great circle.**

The circumference of a great circle is the **circumference** of the sphere.

A great circle divides a sphere into two **hemispheres.**

A baseball can model a sphere. To approximate its surface area, you can take apart its covering. Each of the two pieces suggests a pair of circles with radius r, which is approximately the radius of the ball. The area of the four circles, $4\pi r^2$, suggests the surface area of the ball.



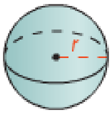
The area of each circle is πr^2.

take note

Theorem 11-10 Surface Area of a Sphere

The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

$$\text{S.A.} = 4\pi r^2$$



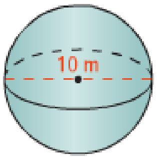
Plan

What are you given?
In sphere problems, make it a habit to note whether you are given the radius or the diameter. In this case, you are given the diameter.

Problem 1 Finding the Surface Area of a Sphere

What is the surface area of the sphere in terms of π?

The diameter is 10 m, so the radius is $\frac{10}{2}$ m, or 5 m.



$\text{S.A.} = 4\pi r^2$	Use the formula for surface area of a sphere.
$= 4\pi(5)^2$	Substitute 5 for *r*.
$= 100\pi$	Simplify.

The surface area is 100π m^2.

Got It? **1.** What is the surface area of a sphere with a diameter of 14 in.? Give your answer in terms of π and rounded to the nearest square inch.

You can use spheres to approximate the surface areas of real-world objects.

Plan

How can you use the length of Earth's equator?
Earth's equator is a great circle that divides Earth into two hemispheres. Its length is Earth's circumference. Use it to find Earth's radius.

Problem 2 Finding Surface Area

Geography **Earth's equator is about 24,902 mi long. What is the approximate surface area of Earth? Round to the nearest thousand square miles.**

Step 1 Find the radius of Earth.

$C = 2\pi r$	Use the formula for circumference.
$24{,}902 = 2\pi r$	Substitute 24,902 for *C*.
$\frac{24{,}902}{2\pi} = r$	Divide each side by 2π.
$r \approx 3963.276393$	Use a calculator.

Step 2 Use the radius to find the surface area of Earth.

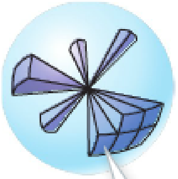
$\text{S.A.} = 4\pi r^2$	Use the formula for surface area.
$= 4\pi$ ANS x^2 enter	Use a calculator. ANS uses the value of *r* from Step 1.
≈ 197387017.5	

The surface area of Earth is about 197,387,000 mi^2.

Got It? **2.** What is the surface area of a melon with circumference 18 in.? Round your answer to the nearest ten square inches.

The following model suggests a formula for the volume of a sphere.

Fill a sphere with a large number n of small pyramids. The vertex of each pyramid is the center of the sphere. The height of each pyramid is approximately the radius r of the sphere. The sum of the areas of the bases of the n pyramids approximates the surface area of the sphere. The sum of the volumes of the n pyramids should approximate the volume of the sphere.



The base of each pyramid touches the surface of the sphere.

$$\text{Volume of each pyramid} = \frac{1}{3}Bh$$

$$\text{Sum of the volumes of } n \text{ pyramids} \approx n \cdot \frac{1}{3}Br$$ Substitute r for h.

$$= \frac{1}{3} \cdot (nB)r$$ Commutative and associative properties of multiplication

$$\approx \frac{1}{3} \cdot (4\pi r^2)r$$ Replace nB with the surface area of a sphere.

$$= \frac{4}{3}\pi r^3$$ Simplify.

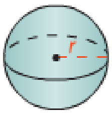
It is reasonable to conjecture that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

take note

Theorem 11-11 Volume of a Sphere

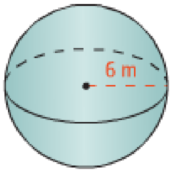
The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$



Problem 3 Finding the Volume of a Sphere

What is the volume of the sphere in terms of π?



$$V = \frac{4}{3}\pi r^3$$ Use the formula for volume of a sphere.

$$= \frac{4}{3}\pi(6)^3$$ Substitute.

$$= 288\pi$$

The volume of the sphere is 288π m^3.

Think

What are the units of the answer? You are cubing the radius, which is in meters (m), so your answer should be in cubic meters (m^3).

Got It? 3. a. A sphere has a diameter of 60 in. What is its volume to the nearest cubic inch?

b. Reasoning Suppose the radius of a sphere is halved. How does this affect the volume of the sphere? Explain.

Notice that you only need to know the radius of a sphere to find its volume and surface area. This means that if you know the volume of a sphere, you can find its surface area.

Problem 4 Using Volume to Find Surface Area

The volume of a sphere is 5000 m^3. What is its surface area to the nearest square meter?

Know	Need	Plan
The volume of a sphere	The radius of the sphere	*Work backward* by using the formula for volume and solving for *r*. Then use the radius to calculate surface area.

Step 1 Find the radius of the sphere.

$V = \frac{4}{3}\pi r^3$ Use the formula for volume of a sphere.

$5000 = \frac{4}{3}\pi r^3$ Substitute.

$5000\left(\frac{3}{4\pi}\right) = r^3$ Solve for r^3.

$\sqrt[3]{5000\left(\frac{3}{4\pi}\right)} = r$ Take the cube root of each side.

$r \approx 10.60784418$ Use a calculator.

Step 2 Find the surface area of the sphere.

$\text{S.A.} = 4\pi r^2$ Use the formula for surface area of a sphere.

$= 4\pi \text{ ANS}$ x^2 enter Use a calculator.

≈ 1414.04792

The surface area of the sphere is about 1414 m^2.

Got It? **4.** The volume of a sphere is 4200 ft^3. What is its surface area to the nearest tenth?

Lesson Check

Do you know HOW?

The diameter of a sphere is 12 ft.

1. What is its surface area in terms of π?

2. What is its volume to the nearest tenth?

3. The volume of a sphere is 80π cm^3. What is its surface area to the nearest whole number?

Do you UNDERSTAND?

4. **Vocabulary** What is the ratio of the area of a great circle to the surface area of the sphere?

5. **Error Analysis** Your classmate claims that if you double the radius of a sphere, its surface area and volume will quadruple. What is your classmate's error? Explain.

Practice and Problem-Solving Exercises

A Practice

Find the surface area of the sphere with the given diameter or radius. Leave your answer in terms of π. **See Problem 1.**

6. $d = 30$ m
7. $r = 10$ in.
8. $d = 32$ mm
9. $r = 100$ yd

Sports Find the surface area of each ball. Leave each answer in terms of π.

10.

$d = 68$ mm

11.

$d = 21$ cm

12.

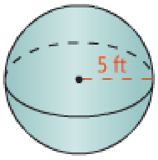
$d = 2\frac{1}{16}$ in.

Use the given circumference to find the surface area of each spherical object. Round your answer to the nearest whole number. **See Problem 2.**

13. a grapefruit with $C = 14$ cm
14. a bowling ball with $C = 27$ in.
15. a pincushion with $C = 8$ cm
16. a head of lettuce with $C = 22$ in.

Find the volume of each sphere. Give each answer in terms of π and rounded to the nearest cubic unit. **See Problem 3.**

17.



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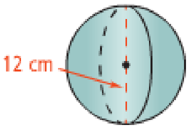

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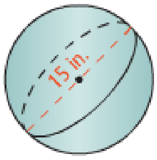

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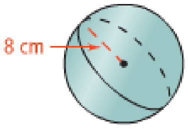

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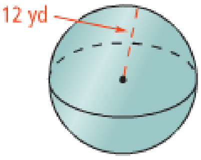

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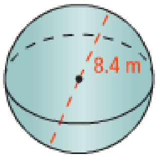


A sphere has the volume given. Find its surface area to the nearest whole number. **See Problem 4.**

23. $V = 900 \text{ in.}^3$
24. $V = 3000 \text{ m}^3$
25. $V = 140 \text{ cm}^3$

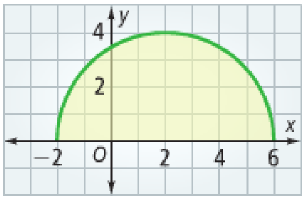
B Apply

26. Mental Math Use $\pi \approx 3$ to estimate the surface area and volume of a sphere with radius 3 cm.

27. Open-Ended Give the dimensions of a cylinder and a sphere that have the same volume.

28. **Visualization** The region enclosed by the semicircle at the right is revolved completely about the x-axis.

a. Describe the solid of revolution that is formed.
b. Find its volume in terms of π.
c. Find its surface area in terms of π.

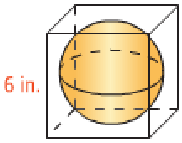


29. **Think About a Plan** A cylindrical tank with diameter 20 in. is half filled with water. How much will the water level in the tank rise if you place a metallic ball with radius 4 in. in the tank? Give your answer to the nearest tenth.

- What causes the water level in the tank to rise?
- Which volume formulas should you use?

30. The sphere at the right fits snugly inside a cube with 6-in. edges. What is the approximate volume of the space between the sphere and cube?

Ⓐ 28.3 in.^{3}
Ⓑ 76.5 in.^{3}
Ⓒ 102.9 in.^{3}
Ⓓ 113.1 in.^{3}



31. **Meteorology** On September 3, 1970, a hailstone with diameter 5.6 in. fell at Coffeyville, Kansas. It weighed about 0.018 lb/in.^{3} compared to the normal 0.033 lb/in.^{3} for ice. About how heavy was this Kansas hailstone?

32. **Reasoning** Which is greater, the total volume of three spheres, each of which has diameter 3 in., or the volume of one sphere that has diameter 8 in.?

33. **Reasoning** How many great circles does a sphere have? Explain.

Find the volume in terms of π of each sphere with the given surface area.

34. 4π m^2
35. 36π in.^{2}
36. 9π ft^2
37. 100π mm^2
38. 25π yd^2
39. 144π cm^2
40. 49π m^2
41. 225π mi^2

42. **Recreation** A spherical balloon has a 14-in. diameter when it is fully inflated. Half of the air is let out of the balloon. Assume that the balloon remains a sphere.

a. Find the volume of the fully inflated balloon in terms of π.
b. Find the volume of the half-inflated balloon in terms of π.
c. What is the diameter of the half-inflated balloon to the nearest inch?

43. **Sports Equipment** The diameter of a golf ball is 1.68 in.

a. Approximate the surface area of the golf ball.
b. **Reasoning** Do you think that the value you found in part (a) is greater than or less than the actual surface area of the golf ball? Explain.

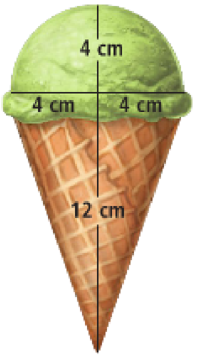
Geometry in 3 Dimensions **A sphere has center (0, 0, 0) and radius 5.**

44. Name the coordinates of six points on the sphere.

45. Tell whether each of the following points is *inside, outside,* or *on the sphere*.

a. $A(0, -3, 4)$
b. $B(1, -1, -1)$
c. $C(4, -6, -10)$

46. **Food** An ice cream vendor presses a sphere of frozen yogurt into a cone, as shown at the right. If the yogurt melts into the cone, will the cone overflow? Explain.

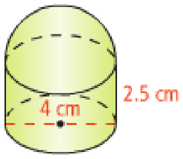


47. The surface area of a sphere is 5541.77 ft^2. What is its volume to the nearest tenth?

48. **Geography** The circumference of Earth at the equator is approximately 40,075 km. About 71% of Earth is covered by oceans and other bodies of water. To the nearest thousand square kilometers, how much of Earth's surface is land?

Find the surface area and volume of each figure.

49.



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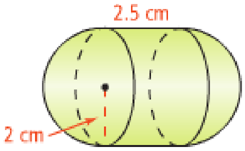

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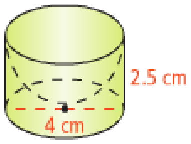
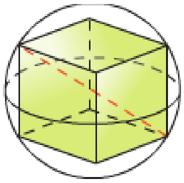


52. **Astronomy** The diameter of the Earth is about 24,902 mi. The diameter of the moon is about 27% of the diameter of Earth. What percent of the volume of Earth is the volume of the moon? Round your answer to the nearest whole percent.

53. **Science** The density of steel is about 0.28 lb/in.^{3}. Could you lift a solid steel ball with radius 4 in.? With radius 6 in.? Explain.

54. A cube with edges 6 in. long fits snugly inside a sphere as shown at the right. The diagonal of the cube is the diameter of the sphere.

- **a.** Find the length of the diagonal and the radius of the sphere. Leave your answer in simplest radical form.
- **b.** What is the volume of the space between the sphere and the cube to the nearest tenth?



Challenge

Find the radius of a sphere with the given property.

55. The number of square meters of surface area equals the number of cubic meters of volume.

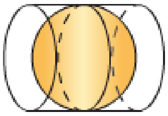
56. The ratio of surface area in square meters to volume in cubic meters is 1 : 5.

57. Suppose a cube and a sphere have the same volume.

- **a.** Which has the greater surface area? Explain.
- **b.** **Writing** Explain why spheres are rarely used for packaging.

58. A plane intersects a sphere to form a circular cross section. The radius of the sphere is 17 cm and the plane comes to within 8 cm of the center. Draw a sketch and find the area of the cross section, to the nearest whole number.

59. **History** At the right, the sphere fits snugly inside the cylinder. Archimedes (about 287–212 B.C.) requested that such a figure be put on his gravestone along with the ratio of their volumes, a finding that he regarded as his greatest discovery. What is that ratio?



Standardized Test Prep

SAT/ACT

60. What is the diameter of a sphere whose surface area is 100π m^2?

Ⓐ 5 m　Ⓑ 10 m　Ⓒ 5π m　Ⓓ 25π m

61. Which of the following statements contradict each other?

I. Opposite sides of $\square ABCD$ are parallel.
II. Diagonals of $\square ABCD$ are perpendicular.
III. $\square ABCD$ is not a rhombus.

Ⓕ I and II　Ⓖ II and III　Ⓗ I and III　Ⓘ none

62. What is the reflection image of (3, 7) across the line $y = 4$?

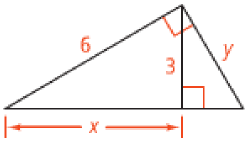
Ⓐ (3, 3)　Ⓑ (−7, 3)　Ⓒ (3, 1)　Ⓓ (3, −7)

63. The radius of a sphere is doubled. By what factor does the surface area of the sphere change?

Ⓕ $\frac{1}{4}$　Ⓖ $\frac{1}{2}$　Ⓗ 2　Ⓘ 4

Short Response

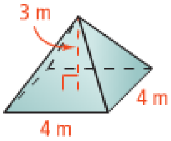
64. Find the values of x and y. Show your work.



Mixed Review

Find the volume of each figure to the nearest cubic unit.　**See Lesson 11-5.**

65.



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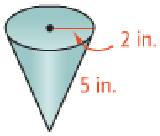

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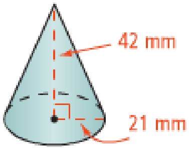


68. A leg of a right triangle has a length of 4 cm and the hypotenuse has a length of 7 cm. Find the measure of each acute angle of the triangle to the nearest degree.　**See Lessons 6-4 and 8-3.**

69. The length of each side of a rhombus is 16. The longer diagonal has length 26. Find the measures of the angles of the rhombus to the nearest degree.

Get Ready! **To prepare for Lesson 11-7, do Exercises 70 and 71.**

Are the figures similar? If so, give the scale factor.　**See Lesson 7-2.**

70. two squares, one with 3-in. sides and the other with 1-in. sides

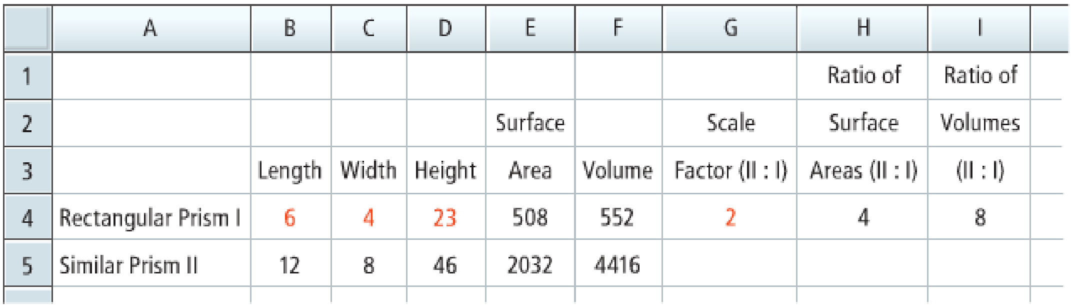
71. two right isosceles triangles, one with a 3-cm hypotenuse and the other with a 1-cm leg

Concept Byte
Use With Lesson 11-7
TECHNOLOGY

Exploring Similar Solids

To explore surface areas and volumes of similar rectangular prisms, you can set up a spreadsheet like the one below. You choose the numbers for length, width, height, and scale factor. The computer uses formulas to calculate all the other numbers.

Activity



	A	B	C	D	E	F	G	H	I
1								Ratio of	Ratio of
2					Surface		Scale	Surface	Volumes
3		Length	Width	Height	Area	Volume	Factor (II : I)	Areas (II : I)	(II : I)
4	Rectangular Prism I	6	4	23	508	552	2	4	8
5	Similar Prism II	12	8	46	2032	4416			

In cell E4, enter the formula =2*(B4*C4+B4*D4+C4*D4). This will calculate the sum of the areas of the six faces of Prism I. In cell F4, enter the formula =B4*C4*D4. This will calculate the volume of Prism I.

In cells B5, C5, and D5 enter the formulas =G4*B4, =G4*C4, and =G4*D4, respectively. These will calculate the dimensions of similar Prism II. Copy the formulas from E4 and F4 into E5 and F5 to calculate the surface area and volume of Prism II.

In cell H4 enter the formula =E5/E4 and in cell I4 enter the formula =F5/F4. These will calculate the ratios of the surface areas and volumes.

Investigate In row 4, enter numbers for the length, width, height, and scale factor. Change the numbers to explore how the ratio of the surface areas and the ratio of the volumes are related to the scale factor.

Exercises

State a relationship that seems to be true about the scale factor and the given ratio.

1. the ratio of volumes

2. the ratio of surface areas

Set up spreadsheets that allow you to investigate the following ratios. State a conclusion from each investigation.

3. the volumes of similar cylinders

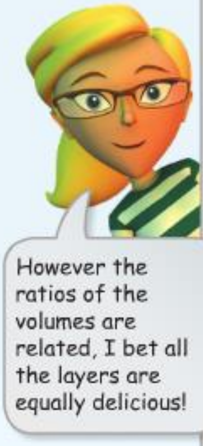
4. the lateral areas of similar cylinders

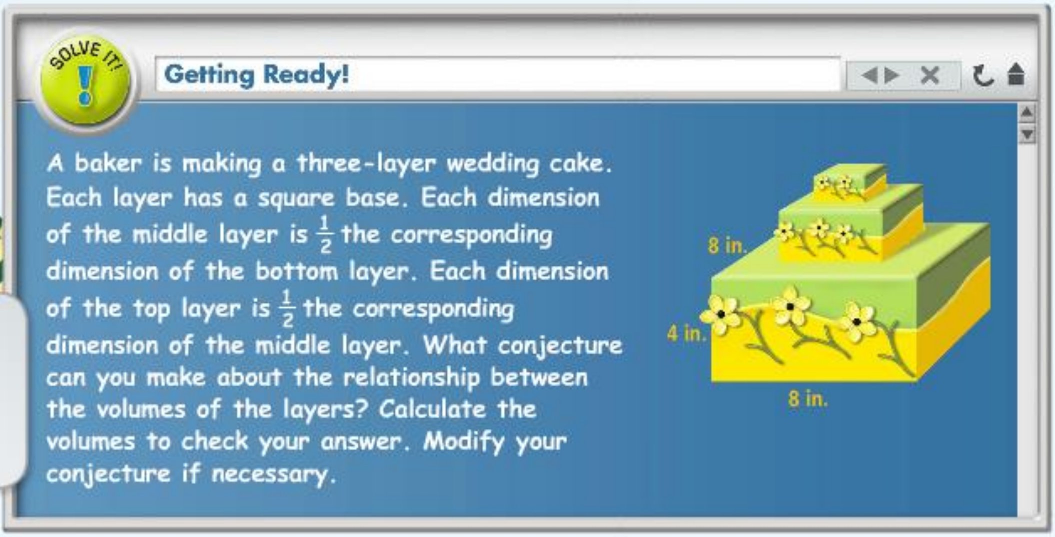
5. the surface areas of similar cylinders

6. the volumes of similar square pyramids

11-7 Areas and Volumes of Similar Solids

Objective To compare and find the areas and volumes of similar solids


However the ratios of the volumes are related, I bet all the layers are equally delicious!



Getting Ready!

A baker is making a three-layer wedding cake. Each layer has a square base. Each dimension of the middle layer is $\frac{1}{2}$ the corresponding dimension of the bottom layer. Each dimension of the top layer is $\frac{1}{2}$ the corresponding dimension of the middle layer. What conjecture can you make about the relationship between the volumes of the layers? Calculate the volumes to check your answer. Modify your conjecture if necessary.

Lesson Vocabulary
- similar solids

Essential Understanding You can use ratios to compare the areas and volumes of similar solids.

Similar solids have the same shape, and all their corresponding dimensions are proportional. The ratio of corresponding linear dimensions of two similar solids is the scale factor. Any two cubes are similar, as are any two spheres.

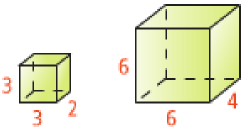
Problem 1 Identifying Similar Solids

Plan

How do you check for similarity? Check that the ratios of the corresponding dimensions are the same. A rectangular prism has three dimensions (ℓ, w, h), so you must check three ratios.

Are the two rectangular prisms similar? If so, what is the scale factor of the first figure to the second figure?

A

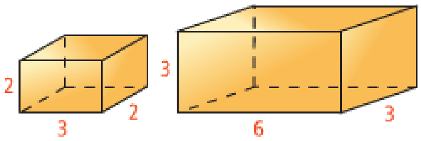


$\frac{3}{6} = \frac{2}{4} = \frac{3}{6}$

The prisms are similar because the corresponding linear dimensions are proportional.

The scale factor is $\frac{1}{2}$.

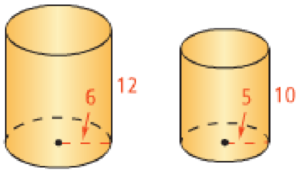
B



$\frac{2}{3} = \frac{2}{3} \neq \frac{3}{6}$

The prisms are not similar because the corresponding linear dimensions are not proportional.

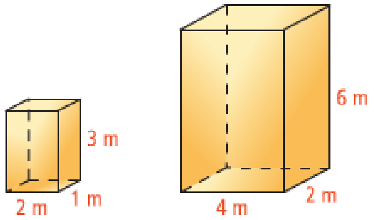
Got It? **1.** Are the two cylinders similar? If so, what is the scale factor of the first figure to the second figure?



The two similar prisms shown here suggest two important relationships for similar solids.

The ratio of the side lengths is 1 : 2.
The ratio of the surface areas is 22 : 88, or 1 : 4.
The ratio of the volumes is 6 : 48, or 1 : 8.

The ratio of the surface areas is the square of the scale factor. The ratio of the volumes is the cube of the scale factor. These two facts apply to all similar solids.



S.A. = 22 m^2
$V = 6\ m^3$

S.A. = 88 m^2
$V = 48\ m^3$

take note

Theorem 11-12 Areas and Volumes of Similar Solids

If the scale factor of two similar solids is $a : b$, then

- the ratio of their corresponding areas is $a^2 : b^2$
- the ratio of their volumes is $a^3 : b^3$

Problem 2 Finding the Scale Factor

The square prisms at the right are similar. What is the scale factor of the smaller prism to the larger prism?

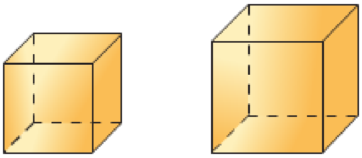
Plan

How can you use the given information? You are given the volumes of two similar solids. Write a proportion using the ratio $a^3 : b^3$.

$\frac{a^3}{b^3} = \frac{729}{1331}$ The ratio of the volumes is $a^3 : b^3$.

$\frac{a}{b} = \frac{9}{11}$ Take the cube root of each side.

The scale factor is 9 : 11.



$V = 729\ cm^3$

$V = 1331\ cm^3$

Got It? **2.** **a.** What is the scale factor of two similar prisms with surface areas 144 m^2 and 324 m^2?

b. **Reasoning** Are any two square prisms similar? Explain.

Problem 3 Using a Scale Factor

Painting **The lateral areas of two similar paint cans are 1019 cm^2 and 425 cm^2. The volume of the smaller can is 1157 cm^3. What is the volume of the larger can?**

Know

- The lateral areas
- The volume of the smaller can

Need

The scale factor

Plan

Use the lateral areas to find the scale factor $a : b$. Then write and solve a proportion using the ratio $a^3 : b^3$ to find the volume of the larger can.

Step 1 Find the scale factor $a : b$.

$$\frac{a^2}{b^2} = \frac{1019}{425}$$ The ratio of the surface areas is $a^2 : b^2$.

$$\frac{a}{b} = \frac{\sqrt{1019}}{\sqrt{425}}$$ Take the positive square root of each side.

Think

Does it matter how you set up the proportion?
Yes. The numerators should refer to the same paint can, and the denominators should refer to the other can.

Step 2 Use the scale factor to find the volume.

$$\frac{V_{\text{large}}}{V_{\text{small}}} = \frac{\left(\sqrt{1019}\right)^3}{\left(\sqrt{425}\right)^3}$$ The ratio of the volumes is $a^3 : b^3$.

$$\frac{V_{\text{large}}}{1157} = \frac{\left(\sqrt{1019}\right)^3}{\left(\sqrt{425}\right)^3}$$ Substitute 1157 for V_{small}.

$$V_{\text{large}} = 1157 \cdot \frac{\left(\sqrt{1019}\right)^3}{\left(\sqrt{425}\right)^3}$$ Solve for V_{large}.

$$V_{\text{large}} \approx 4295.475437$$ Use a calculator.

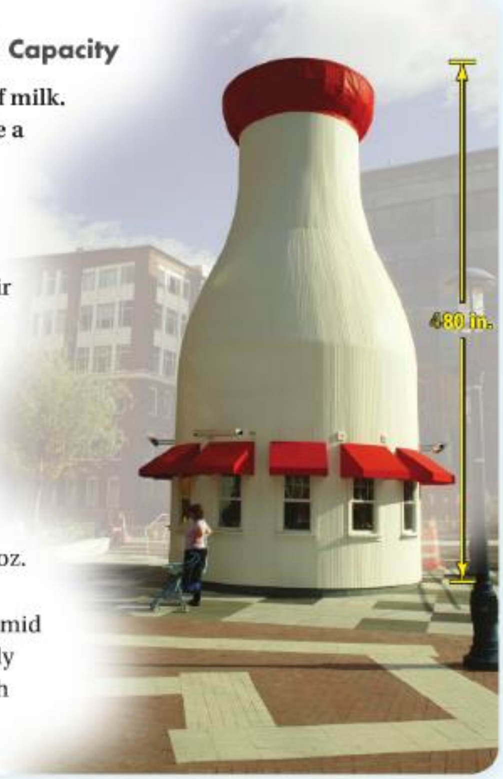
The volume of the larger paint can is about 4295 cm^3.

Got It? **3.** The volumes of two similar solids are 128 m^3 and 250 m^3. The surface area of the larger solid is 250 m^2. What is the surface area of the smaller solid?

You can compare the capacities and weights of similar objects. The capacity of an object is the amount of fluid the object can hold. The capacities and weights of similar objects made of the same material are proportional to their volumes.

Problem 4 Using a Scale Factor to Find Capacity

Containers **A bottle that is 10 in. high holds 34 oz of milk. The sandwich shop shown at the right is shaped like a milk bottle. To the nearest thousand ounces how much milk could the building hold?**



Think

How does capacity relate to volume?
Since the capacities of similar objects are proportional to their volumes, the ratio of their capacities is equal to the ratio of their volumes.

The scale factor of the bottles is 1 : 48.

The ratio of their volumes, and hence the ratio of their capacities, is $1^3 : 48^3$, or 1 : 110,592.

$\frac{1}{110{,}592} = \frac{34}{x}$ Let x = the capacity of the milk-bottle building.

$x = 34 \cdot 110{,}592$ Use the Cross Products Property.

$x = 3{,}760{,}128$ Simplify.

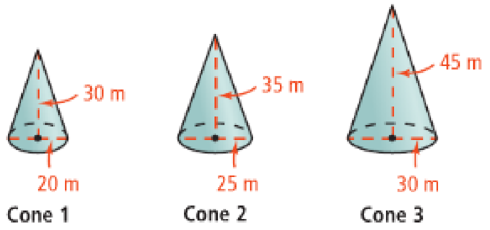
The milk-bottle building could hold about 3,760,000 oz.

Got It? **4.** A marble paperweight shaped like a pyramid weighs 0.15 lb. How much does a similarly shaped marble paperweight weigh if each dimension is three times as large?

Lesson Check

Do you know HOW?

1. Which two of the following cones are similar? What is their scale factor?

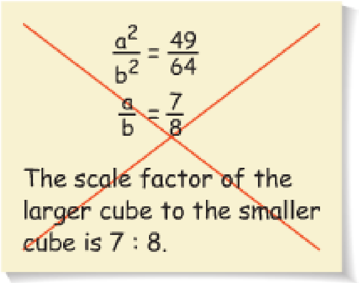


2. The volumes of two similar containers are 115 in.^{3} and 67 in.^{3}. The surface area of the smaller container is 108 in^2. What is the surface area of the larger container?

Do you UNDERSTAND?

3. **Vocabulary** How are similar solids different from similar polygons? Explain.

4. **Error Analysis** Two cubes have surface areas 49 cm^2 and 64 cm^2. Your classmate tried to find the scale factor of the larger cube to the smaller cube. Explain and correct your classmate's error.



$\frac{a^2}{b^2} = \frac{49}{64}$

$\frac{a}{b} = \frac{7}{8}$

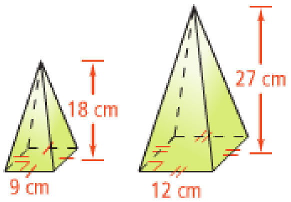
~~The scale factor of the larger cube to the smaller cube is 7 : 8.~~

Practice and Problem-Solving Exercises

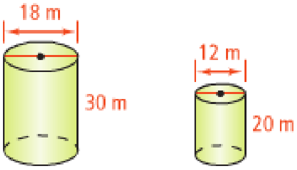
A Practice

For Exercises 5–10, are the two figures similar? If so, give the scale factor of the first figure to the second figure. **See Problem 1.**

5.



6.



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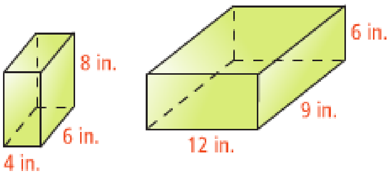

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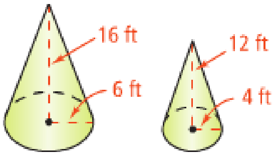
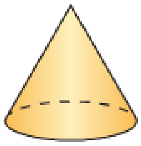


9. two cubes, one with 3-cm edges, the other with 4.5-cm edges

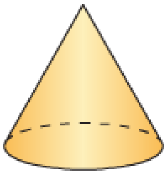
10. a cylinder and a square prism both with 3-in. radius and 1-in. height

Each pair of figures is similar. Use the given information to find the scale factor of the smaller figure to the larger figure. **See Problem 2.**

11.

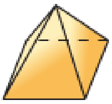


$V = 250\pi$ ft^3

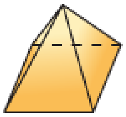


$V = 432\pi$ ft^3

12.

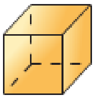


$V = 216$ in.^{3}

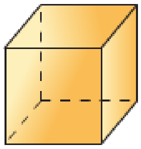


$V = 343$ in.^{3}

13.

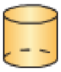


S.A. = 18 m^2

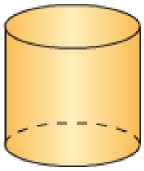


S.A. = 32 m^2

14.



S.A. = 20π yd^2



S.A. = 125π yd^2

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure. **See Problem 3.**

15. S.A. = 248 in.^{2}
S.A. = 558 in.^{2}
V = 810 in.^{3}

16. S.A. = 192 m^2
S.A. = 1728 m^2
V = 4860 m^3

17. S.A. = 52 ft^2
S.A. = 208 ft^2
V = 192 ft^3

The volumes of two similar figures are given. The surface area of the smaller figure is given. Find the surface area of the larger figure.

18. $V = 27\text{ in.}^3$
$V = 125\text{ in.}^3$
$\text{S.A.} = 63\text{ in.}^2$

19. $V = 27\text{ m}^3$
$V = 64\text{ m}^3$
$\text{S.A.} = 63\text{ m}^2$

20. $V = 2\text{ yd}^3$
$V = 250\text{ yd}^3$
$\text{S.A.} = 13\text{ yd}^2$

See Problem 4.

21. Packaging There are 750 toothpicks in a regular-sized box. If a jumbo box is made by doubling all the dimensions of the regular-sized box, how many toothpicks will the jumbo box hold?

22. Packaging A cylinder with a 4-in. diameter and a 6-in. height holds 1 lb of oatmeal. To the nearest ounce, how much oatmeal will a similar 10-in.-high cylinder hold? (*Hint:* 1 lb = 16 oz)

23. Compare and Contrast A regular pentagonal prism has 9-cm base edges. A larger, similar prism of the same material has 36-cm base edges. How does each indicated measurement for the larger prism compare to the same measurement for the smaller prism?

a. the volume
b. the weight

B Apply

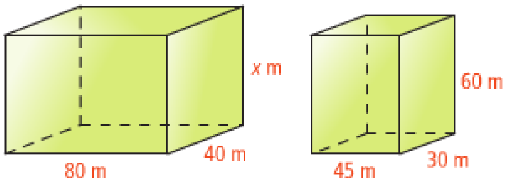
24. Two similar prisms have heights 4 cm and 10 cm.

a. What is their scale factor?
b. What is the ratio of their surface areas?
c. What is the ratio of their volumes?

25. Think About a Plan A company announced that it had developed the technology to reduce the size of its atomic clock, which is used in electronic devices that existing clock made of the same material. The dimensions of the smaller clock will be $\frac{1}{10}$ the dimensions of the company's existing atomic clocks, and it will be $\frac{1}{100}$ the weight. Do these ratios make sense? Explain.

- What is the scale factor of the smaller clock to the larger clock?
- How are the weights of the two objects related to their scale factor?

26. Reasoning Is there a value of x for which the rectangular prisms at the right are similar? Explain.



27. The volume of a spherical balloon with radius 3.1 cm is about 125 cm^3. Estimate the volume of a similar balloon with radius 6.2 cm.

28. Writing Are all spheres similar? Explain.

29. Reasoning A carpenter is making a blanket chest based on an antique chest. Both chests have the shape of a rectangular prism. The length, width, and height of the new chest will all be 4 in. greater than the respective dimensions of the antique. Will the chests be similar? Explain.

30. Two similar pyramids have lateral area 20 ft^2 and 45 ft^2. The volume of the smaller pyramid is 8 ft^3. Find the volume of the larger pyramid.

31. The volumes of two spheres are 729 in.^{3} and 27 in.^{3}.
  a. Find the ratio of their radii.
  b. Find the ratio of their surface areas.

32. The volumes of two similar pyramids are 1331 cm^3 and 2744 cm^3.
  a. Find the ratio of their heights.
  b. Find the ratio of their surface areas.

33. A clown's face on a balloon is 4 in. tall when the balloon holds 108 in.^{3} of air. How much air must the balloon hold for the face to be 8 in. tall?

Copy and complete the table for the similar solids.

	Similarity Ratio	Ratio of Surface Areas	Ratio of Volumes
34.	1 : 2	■ : ■	■ : ■
35.	3 : 5	■ : ■	■ : ■
36.	■ : ■	49 : 81	■ : ■
37.	■ : ■	■ : ■	125 : 512

38. **Literature** In *Gulliver's Travels*, by Jonathan Swift, Gulliver first traveled to Lilliput. The Lilliputian average height was one twelfth of Gulliver's height.
  a. How many Lilliputian coats could be made from the material in Gulliver's coat? (*Hint:* Use the ratio of surface areas.)
  b. How many Lilliputian meals would be needed to make a meal for Gulliver? (*Hint:* Use the ratio of volumes.)

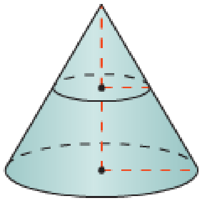
Challenge

39. **Indirect Reasoning** Some stories say that Paul Bunyan was ten times as tall as the average human. Assume that Paul Bunyan's bone structure was proportional to that of ordinary people.
  a. Strength of bones is proportional to the area of their cross section. How many times as strong as the average person's bones would Paul Bunyan's bones be?
  b. Weights of objects made of like material are proportional to their volumes. How many times the average person's weight would Paul Bunyan's weight be?
  c. Human leg bones can support about 6 times the average person's weight. Use your answers to parts (a) and (b) to explain why Paul Bunyan could not exist with a bone structure that was proportional to that of ordinary people.

40. Square pyramids A and B are similar. In pyramid A, each base edge is 12 cm. In pyramid B, each base edge is 3 cm and the volume is 6 cm^3.
  a. Find the volume of pyramid A.
  b. Find the ratio of the surface area of A to the surface area of B.
  c. Find the surface area of each pyramid.

41. A cone is cut by a plane parallel to its base. The small cone on top is similar to the large cone. The ratio of the slant heights of the cones is 1 : 2. Find each ratio.

a. the surface area of the large cone to the surface area of the small cone
b. the volume of the large cone to the volume of the small cone
c. the surface area of the frustum to the surface area of the large cone and to the surface area of the small cone
d. the volume of the frustum to the volume of the large cone and to the volume of the small cone



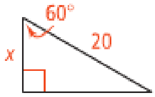
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

42. The slant heights of two similar pyramids are in the ratio 1 : 5. The volume of the smaller pyramid is 60 m^3. What is the volume in cubic meters of the larger pyramid?

43. What is the value of x in the figure at the right?



44. A dilation maps $\triangle JEN$ onto $\triangle J'E'N'$. If $JE = 4.5$ ft, $EN = 6$ ft, and $J'E' = 13.5$ ft, what is $E'N'$ in feet?

45. $\triangle CAR \cong \triangle BUS$, $m\angle C = 25$, and $m\angle R = 39$. What is $m\angle U$?

46. A regular pentagon has a radius of 5 in. What is the area of the pentagon to the nearest square inch?

Mixed Review

47. **Sports Equipment** The circumference of a regulation basketball is between 75 cm and 78 cm. What are the smallest and the largest surface areas that a basketball can have? Give your answers to the nearest whole unit.

See Lesson 11-6.

Find the volume of each sphere to the nearest tenth.

48. diameter = 6 in.

49. circumference = 2.5π m

50. radius = 6 in.

51. The altitude to the hypotenuse of right $\triangle ABC$ divides the hypotenuse into 12-mm and 16-mm segments. Find the length of each of the following.

See Lesson 7-4.

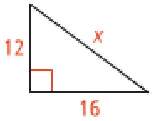
a. the altitude to the hypotenuse
b. the shorter leg of $\triangle ABC$
c. the longer leg of $\triangle ABC$

Get Ready! **To prepare for Lesson 12-1, do Exercises 52–54.**

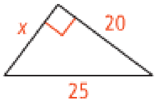
Find the value of x.

See Lesson 8-1.

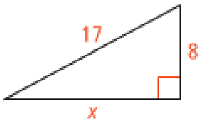
52.



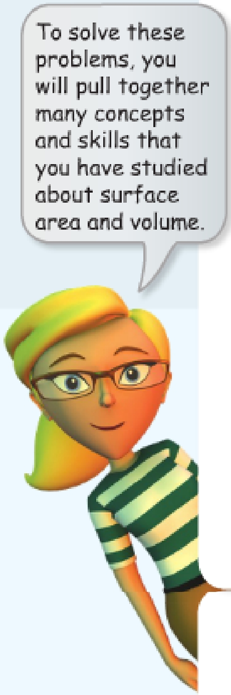
53.



54.



11 *Pull It* All Together

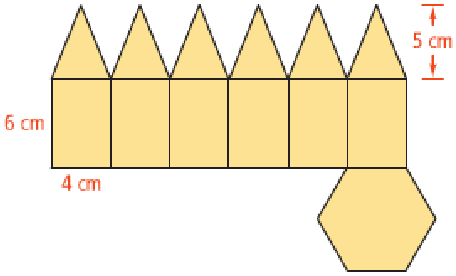

To solve these problems, you will pull together many concepts and skills that you have studied about surface area and volume.

BIG idea Visualization

You can use a net to visualize a three-dimensional solid.

Task 1

The net of a composite space figure is shown below.



a. What figures make up the composite space figure?

b. What is the surface area of the composite space figure? Round your answer to the nearest square centimeter.

BIG idea Measurement

You can use formulas to find the volumes of various solids.

Task 2

Jack and Maureen are opening an ice hotel. The structure and furniture are made out of ice. They must determine the weight of some of the ice furniture to decide the size of the crane needed to lift each piece into place. Maureen knows that ice weighs approximately 62.4 pounds per cubic foot. Find the approximate weight of each of the figures below.

a. The stage in the banquet hall measures 22 ft by 15 ft by 4 ft.

b. Two cylindrical pillars will flank the front door. Each cylinder has diameter 1 ft and height 9 ft.

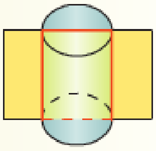
c. A series of four spheres will decorate the reception area. The largest sphere has diameter 4 ft. The other three spheres have diameter 2 ft, 1 ft, and $\frac{1}{2}$ ft.

11 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Visualization
You can determine the intersection of a solid and a plane by visualizing how the plane slices the solid to form a two-dimensional cross section.

Space Figures and Cross Sections (Lesson 11-1)
This vertical plane intersects the cylinder in a rectangular cross section.



Surface Areas and Volumes of Prisms, Cylinders, Pyramids, and Cones (Lessons 11-2 through 11-5)

	Surface Area (S.A.)	Volume (V)
Prism	$ph + 2B$	Bh
Cylinder	$2\pi rh + 2B$	Bh
Pyramid	$\frac{1}{2}p\ell + B$	$\frac{1}{3}Bh$
Cone	$\pi r\ell + B$	$\frac{1}{3}Bh$

2 Measurement
You can find the surface area or volume of a solid by first choosing a formula to use and then substituting the needed dimensions into the formula.

Surface Areas and Volumes of Spheres (Lesson 11-6)

$\text{S.A.} = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

3 Similarity
The surface areas of similar solids are proportional to the squares of their corresponding dimensions. The volumes are proportional to the cubes of their corresponding dimensions.

Areas and Volumes of Similar Solids (Lesson 11-7)
If the scale factor of two similar solids is $a : b$, then

- the ratio of their areas is $a^2 : b^2$
- the ratio of their volumes is $a^3 : b^3$

Chapter Vocabulary

- altitude (pp. 699, 701, 708, 711)
- center of a sphere (p. 733)
- cone (p. 711)
- cross section (p. 690)
- cylinder (p. 701)
- edge (p. 688)
- face (p. 688)
- great circle (p. 733)
- hemisphere (p. 733)
- lateral area (pp. 700, 702, 709, 711)
- lateral face (pp. 699, 708)
- polyhedron (p. 688)
- prism (p. 699)
- pyramid (p. 708)
- right cone (p. 711)
- right cylinder (p. 701)
- right prism (p. 699)
- slant height (pp. 708, 711)
- sphere (p. 733)
- surface area (pp. 700, 702, 709, 711)
- volume (p. 717)

Choose the correct term to complete each sentence.

1. A set of points in space equidistant from a given point is called a(n) __?__.

2. A(n) __?__ is a polyhedron in which one face can be any polygon and the lateral faces are triangles that meet at a common vertex.

3. If you slice a prism with a plane, the intersection of the prism and the plane is a(n) __?__ of the prism.

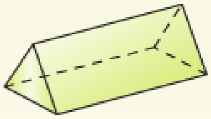
11-1 Space Figures and Cross Sections

Quick Review

A **polyhedron** is a three-dimensional figure whose surfaces are polygons. The polygons are **faces** of the polyhedron. An **edge** is a segment that is the intersection of two faces. A **vertex** is a point where three or more edges intersect. A **cross section** is the intersection of a solid and a plane.

Example

How many faces and edges does the polyhedron have?



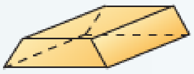
The polyhedron has 2 triangular bases and 3 rectangular faces for a total of 5 faces.

The 2 triangles have a total of 6 edges. The 3 rectangles have a total of 12 edges. The total number of edges in the polyhedron is one half the total of 18 edges, or 9.

Exercises

Draw a net for each three-dimensional figure.

4.



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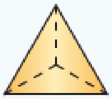


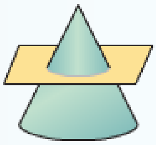
Use Euler's Formula to find the missing number.

6. $F = 5, V = 5, E = ■$

7. $F = 6, V = ■, E = 12$

8. How many vertices are there in a solid with 4 triangular faces and 1 square base?

9. Describe the cross section in the figure at the right.



10. Sketch a cube with an equilateral triangle cross section.

11-2 Surface Areas of Prisms and Cylinders

Quick Review

The **lateral area of a right prism** is the product of the perimeter of the base and the height. The **lateral area of a right cylinder** is the product of the circumference of the base and the height of the cylinder. The **surface area** of each solid is the sum of the lateral area and the areas of the bases.

Example

What is the surface area of a cylinder with radius 3 m and height 6 m? Leave your answer in terms of π.

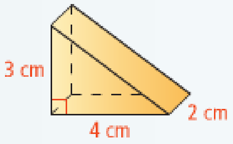
$\text{S.A.} = \text{L.A.} + 2B$	Use the formula for surface area of a cylinder.
$= 2\pi rh + 2(\pi r^2)$	Substitute formulas for lateral area and area of a circle.
$= 2\pi(3)(6) + 2\pi(3)^2$	Substitute 3 for *r* and 6 for *h*.
$= 36\pi + 18\pi$	Simplify.
$= 54\pi$	

The surface area of the cylinder is 54π m^2.

Exercises

Find the surface area of each figure. Leave your answers in terms of π where applicable.

11.



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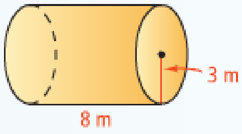
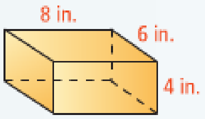


13.



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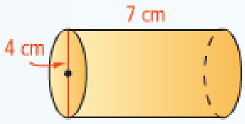

15. A cylinder has radius 2.5 cm and lateral area 20π cm^2. What is the surface area of the cylinder in terms of π?

11-3 Surface Areas of Pyramids and Cones

Quick Review

The **lateral area of a regular pyramid** is half the product of the perimeter of the base and the slant height. The **lateral area of a right cone** is half the product of the circumference of the base and the slant height. The **surface area** of each solid is the sum of the lateral area and the area of the base.

Example

What is the surface area of a cone with radius 3 in. and slant height 10 in.? Leave your answer in terms of π.

$\text{S.A.} = \text{L.A.} + B$	Use the formula for surface area of a cone.
$= \pi r \ell + \pi r^2$	Substitute formulas for lateral area and area of a circle.
$= \pi(3)(10) + \pi(3)^2$	Substitute 3 for r and 10 for ℓ.
$= 30\pi + 9\pi$	Simplify.
$= 39\pi$	

The surface area of the cone is 39π in.^{2}.

Exercises

Find the surface area of each figure. Round your answers to the nearest tenth.

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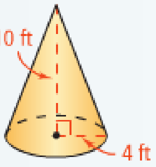

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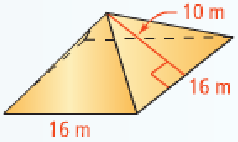

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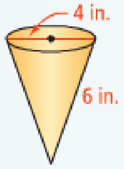

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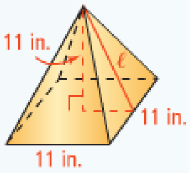


20. Find the formula for the base area of a prism in terms of surface area and lateral area.

11-4 and 11-5 Volumes of Prisms, Cylinders, Pyramids, and Cones

Quick Review

The **volume** of a space figure is the space that the figure occupies. Volume is measured in cubic units. The **volume of a prism** and the **volume of a cylinder** are the product of the area of a base and the height of the solid. The **volume of a pyramid** and the **volume of a cone** are one third the product of the area of the base and the height of the solid.

Example

What is the volume of a rectangular prism with base 3 cm by 4 cm and height 8 cm?

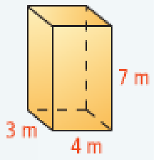
$V = Bh$	Use the formula for the volume of a prism.
$= (3 \cdot 4)(8)$	Substitute.
$= 96$	Simplify.

The volume of the prism is 96 cm^3.

Exercises

Find the volume of each figure. If necessary, round to the nearest tenth.

21.



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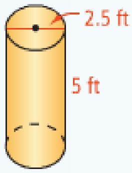

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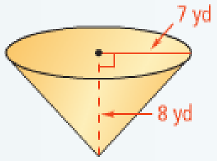

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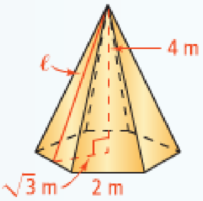


11-6 Surface Areas and Volumes of Spheres

Quick Review

The **surface area of a sphere** is four times the product of π and the square of the radius of the sphere. The **volume of a sphere** is $\frac{4}{3}$ the product of π and the cube of the radius of the sphere.

Example

What is the surface area of a sphere with radius 7 ft? Round your answer to the nearest tenth.

$\text{S.A.} = 4\pi r^2$ Use the formula for surface area of a sphere.

$= 4\pi(7)^2$ Substitute.

≈ 615.8 Simplify.

The surface area of the sphere is about 615.8 ft^2.

Exercises

Find the surface area and volume of a sphere with the given radius or diameter. Round your answers to the nearest tenth.

25. $r = 5$ in.

26. $d = 7$ cm

27. $d = 4$ ft

28. $r = 0.8$ ft

29. What is the volume of a sphere with a surface area of 452.39 cm^2? Round your answer to the nearest hundredth.

30. What is the surface area of a sphere with a volume of 523.6 m^3? Round your answer to the nearest square meter.

31. **Sports Equipment** The circumference of a lacrosse ball is 8 in. Find its volume to the nearest tenth of a cubic inch.

11-7 Areas and Volumes of Similar Solids

Quick Review

Similar solids have the same shape and all their corresponding dimensions are proportional.

If the scale factor of two similar solids is $a : b$, then the ratio of their corresponding surface areas is $a^2 : b^2$, and the ratio of their volumes is $a^3 : b^3$.

Example

Is a cylinder with radius 4 in. and height 12 in. similar to a cylinder with radius 14 in. and height 35 in.? If so, give the scale factor.

$\frac{4}{14} \neq \frac{12}{35}$

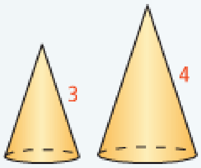
The cylinders are not similar because the ratios of corresponding linear dimensions are not equal.

Exercises

32. **Open-Ended** Sketch two similar solids whose surface areas are in the ratio 16 : 25. Include dimensions.

For each pair of similar solids, find the ratio of the volume of the first figure to the volume of the second.

33.



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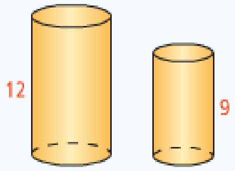

35. **Packaging** There are 12 pencils in a regular-sized box. If a jumbo box is made by tripling all the dimensions of the regular-sized box, how many pencils will the jumbo box hold?

11 Chapter Test

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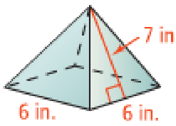
Do you know HOW?

Draw a net for each figure. Label the net with appropriate dimensions.

1.



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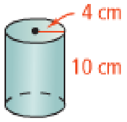
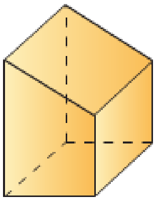


Use the polyhedron at the right for Exercises 3 and 4.



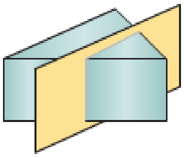
3. Verify Euler's Formula for the polyhedron.

4. Draw a net for the polyhedron. Verify $F + V = E + 1$ for the net.

5. What is the number of edges in a pyramid with seven faces?

Describe the cross section formed in each diagram.

6.



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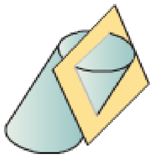
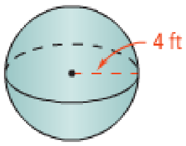


8. **Aviation** The flight data recorders on commercial airlines are rectangular prisms. The base of a recorder is 15 in. by 8 in. Its height ranges from 15 in. to 22 in. What are the largest and smallest possible volumes for the recorder?

Find the volume and surface area of each figure to the nearest tenth.

9.



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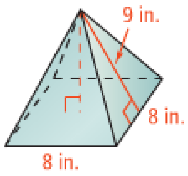

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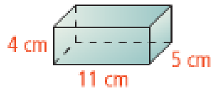

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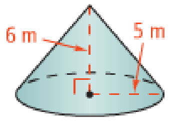

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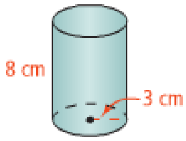

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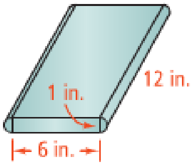


15. List these solids in order from the one with least volume to the one with the greatest volume.

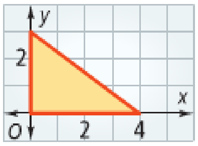
A. a cube with edge 5 cm
B. a cylinder with radius 4 cm and height 4 cm
C. a square pyramid with base edges 6 cm and height 6 cm
D. a cone with radius 4 cm and height 9 cm
E. a rectangular prism with a 5 cm-by-5 cm base and height 6 cm

16. **Painting** The floor of a bedroom is 12 ft by 15 ft and the walls are 7 ft high. One gallon of paint covers about 450 ft^2. How many gallons of paint do you need to paint the walls of the bedroom?

Do you UNDERSTAND?

17. **Reasoning** What solid has a cross section that could either be a circle or a rectangle?

18. **Visualization** The triangle is revolved completely about the y-axis.



a. Describe the solid of revolution that is formed.
b. Find its lateral area and volume in terms of π.

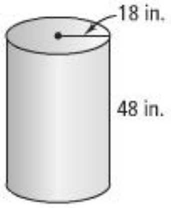
19. **Open-Ended** Draw two different solids that have volume 100 in.^3. Label the dimensions of each solid.

11 Cumulative Test Prep

TIPS FOR SUCCESS

Some questions ask you to use a formula to estimate volume. Read the sample question at the right. Then follow the tips to answer it.

The tank below is filled with gasoline. Which is closest to the volume of the tank in cubic feet?



Ⓐ 28 ft^3

Ⓑ 864 ft^3

Ⓒ 4072 ft^3

Ⓓ 48,858 ft^3

TIP 1

Check whether the units in the problem match the units in the answer choices. You may need to convert measurements.

TIP 2

Identify the given solid and what you want to find so you can select the correct formula.

Think It Through

Convert the radius and the height given in the figure to feet: $r = 1.5$ ft and $h = 4$ ft. Substitute the values for r and h into the formula for the volume of a cylinder and simplify: $V = \pi r^2 h = \pi(1.5)^2(4) \approx 28$. The correct answer is A.

Vocabulary Builder

As you solve problems, you must understand the meanings of mathematical terms. Choose the correct term to complete each sentence.

A. The bases of a cylinder are (*circles, polygons*).

B. An arc of a circle that is larger than a semicircle is a (*major arc, minor arc*).

C. Each polygon of a polyhedron is called a(n) (*edge, face*).

D. A (*hemisphere, great circle*) is the intersection of a plane and a sphere through the center of the sphere.

E. The length of the altitude of a pyramid is the (*height, slant height*) of the pyramid.

F. The area of a net of a polyhedron is equal to the (*lateral area, surface area*) of the polyhedron.

G. In a parallelogram, one diagonal is always the (*perpendicular bisector, segment bisector*) of the other diagonal.

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. A pyramid has a volume of 108 m^3. A similar pyramid has base edges and a height that are $\frac{1}{3}$ those of the original pyramid. What is the volume of the second pyramid?

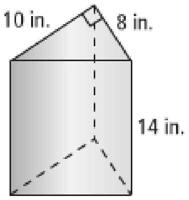
Ⓐ 3 m^3

Ⓑ 4 m^3

Ⓒ 12 m^3

Ⓓ 36 m^3

2. What is the volume of the triangular prism at the right?



Ⓕ 520 in.^{3}

Ⓖ 560 in.^{3}

Ⓗ 600 in.^{3}

Ⓘ 1120 in.^{3}

3. Which quadrilateral CANNOT have one diagonal that bisects the other?

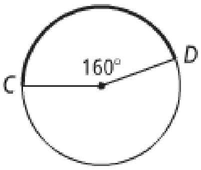
Ⓐ square

Ⓑ trapezoid

Ⓒ kite

Ⓓ parallelogram

4. The diameter of the circle is 12 units. What is the minor arc length from point C to point D?



(F) $\frac{16}{3}\pi$ units

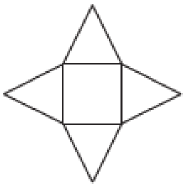
(G) 8π units

(H) 16π units

(I) $\frac{160}{3}\pi$ units

5. Which is a net for a pentagonal pyramid?

(A)



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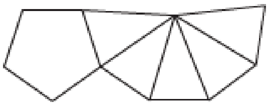
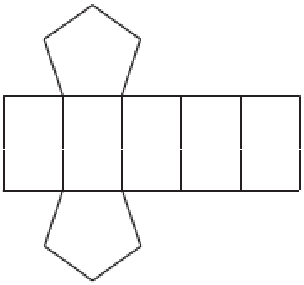
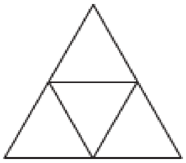


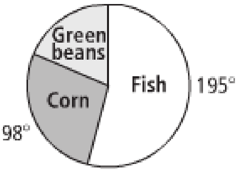
(C)



(D)



6. A frozen dinner is divided into three sections on a circular plate with a 12-in. diameter. What is the approximate arc length of the section containing green beans?



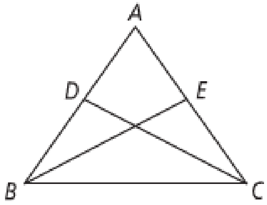
(F) 7 in.

(G) 10 in.

(H) 16 in.

(I) 20 in.

7. Suppose $\overline{AD} \cong \overline{AE}$, $\overline{DB} \cong \overline{EC}$, and $\angle ABC \cong \angle ACB$. Which statement must be proved first to prove $\triangle ABE \cong \triangle ACD$ by SSS?



(A) $\angle A \cong \angle A$

(B) $\overline{AB} \cong \overline{BC}$

(C) $\overline{AC} \cong \overline{AC}$

(D) $\overline{DC} \cong \overline{EB}$

8. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Which statement is true?

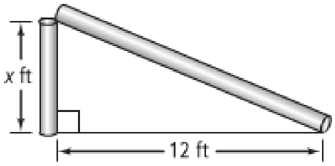
(F) The volume depends on the mean of π, the radius, and the height.

(G) The volume depends only on the square of the radius.

(H) The radius depends on the product of the height and π.

(I) The volume depends on both the height and the radius.

9. A wooden pole was broken during a windstorm. Before the pole broke, the height of the pole above the ground was 18 ft. When it broke, the top of the pole hit the ground 12 ft from the base.



How tall is the part of the pole that remained standing?

(A) 5 ft

(B) 10 ft

(C) 13 ft

(D) 15 ft

10. $\triangle ABC$ is a right isosceles triangle. Which statement CANNOT be true?

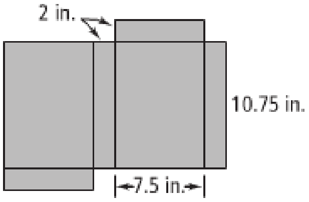
(F) The length of the hypotenuse is $\sqrt{2} \cdot$ the length of a leg.

(G) $AB = BC$

(H) The hypotenuse is the shortest side of the triangle.

(I) $m\angle B = 90$

GRIDDED RESPONSE

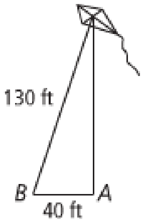


11. The net of a cereal box is shown at the right. What is the surface area in square inches?

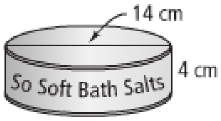
12. A polyhedron has 10 vertices and 7 faces. How many edges does the polyhedron have?

13. The radius of a sphere with a volume of 2 m^3 is quadrupled. What is the volume of the new sphere in cubic meters?

14. Molly stands at point *A* directly below a kite that Kat is flying from point *B*. Kat has let out 130 ft of the string. Molly stands 40 ft from Kat. To the nearest foot, how many feet from the ground is the kite?

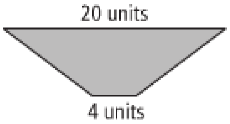


15. The figure below shows a cylindrical tin of bath salts. To the nearest square centimeter, what is the surface area of the tin?



16. A regular octagon has a side length of 8 m and an approximate area of 309 m^2. The side length of another regular octagon is 4 m. To the nearest square meter, what is the approximate area of the smaller octagon?

17. The lengths of the bases of an isosceles trapezoid are shown below. If the perimeter of this trapezoid is 44 units, what is its area in square units?



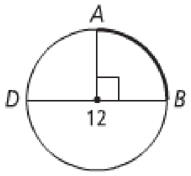
18. What is the surface area, in square centimeters, of a rectangular prism that is 9 cm by 8 cm by 10 cm?

Short Response

19. A great circle of a sphere has a circumference of 3π cm. What is the volume of the sphere in terms of π? Show your work.

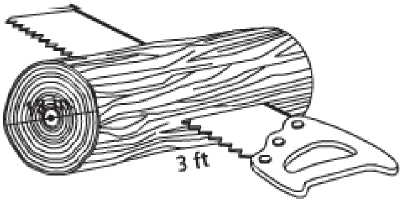
20. Anna has cereal in a cylindrical container that has a diameter of 6 in. and a height of 12 in. She wants to store the cereal in a rectangular prism container. What is one possible set of dimensions for a rectangular prism container that would be close to the volume of the cylindrical container? Show your work.

21. What is the length of $\widehat{AB}$ in terms of π? Show your work.



Extended Response

22. A log is cut lengthwise through its center. To the nearest square inch, what is the surface area of one of the halves? Explain your reasoning.



23. When airplane pilots make a visual sighting of an object outside the airplane, they often refer to the dial of a clock to help locate the object. For example, an object at 12 o'clock is straight ahead, an object at 3 o'clock is 90° to the right, and so on.

Suppose that two pilots flying two airplanes in the same direction spot the same object. Pilot A reports the object at 12 o'clock, and Pilot B reports the object at 2 o'clock. At the same time, Pilot A reports seeing the other airplane at 9 o'clock.

Draw a diagram showing the possible locations of the two planes and the object. If the planes are 800 ft apart, how far is Pilot A from the object? Show your work.